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A Resolution of the Classical Wave-Particle Problem

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The classical wave-particle problem is resolved in accord with Newton's concept of the particle nature of light by associating particle density and flux with the classical wave energy density and flux. Point particles flowing along discrete trajectories yield interference and diffraction patterns, as illustrated by Young's double pinhole interference. Bound particle motion is prescribed by standing waves. Particle motion as a function of time is presented for the case of a "particle in a box." Initial conditions uniquely determine the subsequent motion. Some discussion regarding quantum theory is preseted.

1. HISTORICAL BACKGROUND

Newton's[(1)] extensive experimental investigations into the "wave" and the particle nature of light provide even today the most worthwhile and penetrating insight into the actual physical problems involved. Young[(2)] performed his double pinhole experiment, using Newton's values for the wavelength of light, in order to support Newton's views of light. Young was surprised when he was attacked for suggesting ideas counter to Newton. Fresnel[(3)] developed the mathematics of Young's superposition principle, which Young proposed after being inspired by certain ideas of Newton. Ironically, Young's double pinhole experiment came to be interpreted as evidence that light was a "wave" rather than a flux of particles. Hamilton[(4)] in 1834, following Maupertuis,[(5)] tried to resolve the wave-particle problem by showing that particle flow could yield *approximate* "wave" behavior in the geometrical optics limit. A prescription of the trajectories that could

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yield *exact* "wave" behavior was not presented in the nineteenth century, despite the fact that all the necessary theoretical and mathematical apparatus to do so (see below) was developed before 1880.

At the turn of the century Planck[6] showed that light was quantized in units of energy $\hbar\omega$. Einstein[7] suggested that light might consist of particles to explain the photoelectric effect. In 1923 de Broglie[8] suggested that particles might behave as "waves" and was able to specify the correct propagation constant, $\mathbf{k} = \mathbf{p}/\hbar$. Schrödinger[9] then showed how one could use wave theory to generate the correct energy eigenvalues for atomic systems. Unfortunately, he based his ideas upon Hamilton's *approximate* model, which has perpetuated difficulties when *exact* answers are sought. In 1926 Madelung[10] and de Broglie,[11] recognizing the need for an *exact* resolution to the wave-particle problem, proposed an *ad hoc* method for obtaining discrete particle trajectories, a method rediscovered by Bohm[12] in 1952. The present author[13,14] was also guilty of a similar proposal.

In 1976 Prosser[15] published an excellent paper indicating the fact that the underlying causal reality for the formation of interference and diffraction patterns is the energy flow, as given by the Poynting's vector for the case of light. He presented diagrams of the energy flow by a knife edge and through two finite-width slits. Because of the large width of the slits in comparison to the distance between them and because only the immediate neighborhood of one of the slits is shown, his Figure 3c does not reveal the flow that would normally be expected from Young's double-slit interference. The flow (also given by the Poynting's vector), represented here in Fig. 1 for two *pinholes* of zero size, is more characteristic. In a paper immediately following the first, Prosser[16] speculated about possible "wave packets," which, while extending throughout all of space, could somehow appear localized as point particles when convenient. He suggested an experiment in which the two slits are to be opened sequentially in time. Prosser's theory, however, being only a time-average theory, cannot predict the transient wave behavior that would arise.

Philippidis, Dewdney, and Hiley[17] in 1979 presented the trajectories to be expected for two slits using the *ad hoc* prescription for particle trajectories proposed by Madelung, de Broglie, and Bohm. While this paper presents an excellent review of some of the philosophical questions and indicates the need to resolve the wave-particle problem using discrete particle trajectories, it presents an artificial flow pattern which does not fit the experimental facts. The actual observed flow follows the classical Poynting's vector as given by Prosser[15] or the present author (below). There are certain qualitative similarities between their flow pattern and the actual flow pattern; but in order to be precisely correct their result would have to reduce mathematically to the Poynting's vector, which it does not. In order to predict the observed classical "wave" results, an adequate particle theory

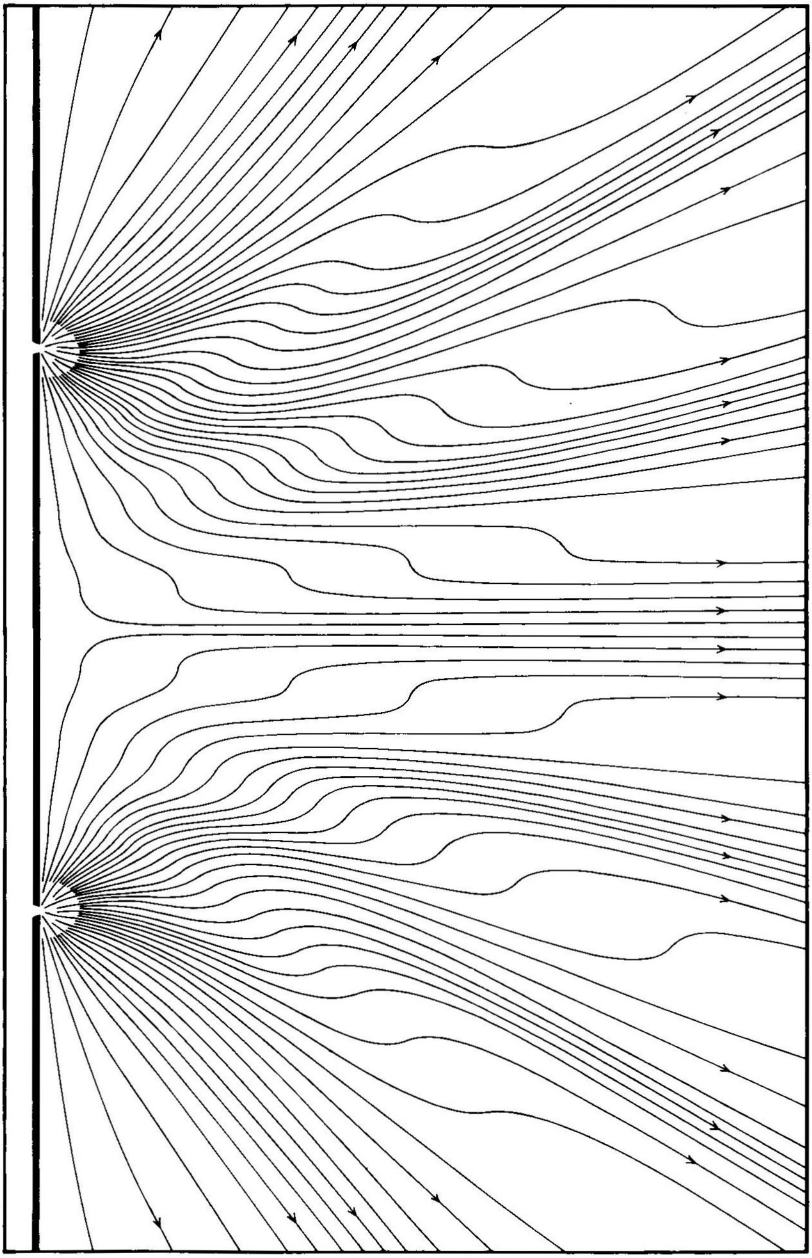

Fig. 1. Particle trajectories which yield Young's double pinhole interference, where the distance between pinholes is 3.6 times the wavelength, Eqs. (4) and (15).

must reduce to precisely the same mathematical expressions as given by the classical wave theory.

The interesting paper by Hirschfelder and Christoph[18] involving reflection and penetration by a barrier, although also based upon the Madelung–de Broglie–Bohm theory, presents many valuable ideas and references. The more detailed analysis of Dewdney and Hiley[19] for reflection and penetration of "wave packets" by a barrier, using the Bohm[12] theory, also fails to yield classical expressions and, thus, also fails to be compatible with macroscopic observations.

2. THE SUPERPOSITION PARADOX OF CLASSICAL WAVE THEORY

It needs to be stressed here that the ordinary classical wave theory, as usually conceived, is not free of basic difficulties. A desired solution to the wave equation may be frequently represented as a superposition of more elementary solutions. Classically it is customary to assign a causal significance to this superposition. For example, in the Young's double-pinhole experiment component waves propagated along straight lines from each of the two pinholes combine by addition of their amplitudes at the point of observation to *cause* the resultant amplitude observed. The addition of the component waves is certainly mathematically correct; and, in addition, it yields a prediction of the amplitude observed. But, unfortunately, it also leads to the following difficulty.

There is no experimental evidence that anything physical is actually propagated from each of the pinholes to the point of observation. In fact, there appears to be ample evidence to the contrary. A straight line drawn from one of the pinholes to a point of observation on the other side of the midline far from the pinholes crosses surfaces of zero intensity or regions of zero energy flow. The paradox then arises as to how an active effect can be propagated across a surface where it is never observed.

When the classical energy flux is calculated, it is seen (Fig. 1) that no energy flows across a surface of zero intensity. Today it is recognized that an active effect, involving an irreversible consequence, requires a cause that can supply energy. All of the light energy that develops a photographic plate, ejects photoelectrons, heats a thermal detector, or causes any other such irreversible effect is seen to be propagated from one pinhole only and not from both. It must, therefore, be concluded that the classical superposition principle, while mathematically correct and while providing an adequate prediction of the observed energy flux, does not represent a *causal* principle.

It should be noted that a principle that provides predictions does not necessarily imply causality. Just because one can predict the rising of the sun from the rooster's crow does not mean that the rooster causes the sun to rise. The rooster does not physically affect the earth's rotation. The failure of the one pinhole to contribute energy to the point of observation similarly indicates that the one pinhole cannot play an active role in activating a detector.

It may then be asked: Why is a two-pinhole pattern different from a single-pinhole pattern? The pinhole that supplies no energy must clearly play a role; but it is merely a passive role. For example, a train travels around a mountain because the tracks are laid around the mountain. The tracks play a passive role. The train itself, however, can *cause* irreversible effects, since it carries kinetic energy. The pinhole that supplies no energy must help to establish a passive static or steady-state field which guides the energy along the appropriate path to produce the observed two-pinhole interference pattern. It must be concluded that the superposition principle is merely a mathematical convenience devoid of any direct physical or causal significance.

3. SPECIFICATION OF PARTICLE MOTION YIELDING "WAVE" BEHAVIOR

The motion of point particles yielding all observed "wave" behavior *exactly*, including interference and diffraction, can be specified by associating the particle density ρ with the wave energy density ε and the particle flux $\mathbf{J}$ with the wave energy flux $\mathbf{S}$; thus,

$$\rho = \kappa\varepsilon, \qquad \mathbf{J} = \kappa\mathbf{S} \tag{1}$$

where κ is an appropriate proportionality constant. If it is assumed that *all* of the energy and momentum flux normally ascribed to a "wave" is actually carried by the particles, then this association, Eq. (1), becomes necessary and unique. This trivial resolution of the classical wave-particle problem might have been presented as soon as an energy density and flux came to be associated with a classical wave. Perhaps the resolution has, in fact, been mentioned; but it has not apparently been heretofore used to solve any specific problems.

The requisite particle motion may be obtained by integrating the instantaneous particle velocity $\mathbf{w}$ given by

$$\mathbf{w} = d\mathbf{r}/dt = \mathbf{J}/\rho = S/\varepsilon \tag{2}$$

where **r** is the position of a particle at the time t. From classical scalar wave theory, ε and **S** are given uniquely by

$$\begin{aligned} \varepsilon &= (1/2)[(\nabla\Psi)^2 + (\partial\Psi/\partial t)^2/u^2] \\ \mathbf{S} &= -\nabla\Psi\,\partial\Psi/\partial t \end{aligned} \tag{3}$$

where u is the phase velocity and Ψ is a solution to the scalar wave equation. An appropriate amplitude factor is assumed to be absorbed into Ψ in order to make the dimensionality of Eqs. (3) correct.

Since classical wave theory is completely predictable, the wave function Ψ must, in general, be taken as pure real (or identically reducible to a pure real expression). For example, the displacement y of a violin string in millimeters at a particular instant t, given as a certain number of milliseconds from $t = 0$, must be a real number. The only admissible form to predict this real number precisely is then the real expression

$$y = A\sin(\omega t + \phi_0)$$

where A is the amplitude, ω is the angular frequency, and ϕ_0 is a phase constant. It may also be noted that pure real notation is necessary to satisfy precise initial conditions (as well as boundary conditions). The complex expression

$$y = A\exp(i\omega t)$$

is ambiguous, since two real numbers are involved. Generally complex notation is used only when time averages are involved and precise prediction is *not* required. For time-average quantities it is not necessary to know the precise phase. In general, complex notation becomes cumbersome and unwieldy when exact predictions are required. When perfectly general solutions are desired, complex notation should be avoided; it can lead to the possibility of error. For example, Schrödinger realized he had committed an error with his use of complex notation; but he had gone too far and could no longer extricate himself. The imaginary number i became an integral and inexcusable part of the traditional quantum theory.

It should be noted that the present scalar wave theory is also appropriate for light; since time-harmonic solutions to Maxwell's equations can be expressed in terms of scalar waves. It is for this reason that beginning courses in physical optics are primarily limited to scalar wave theory. It is thus possible to say for the case of light that the second of Eqs. (3) is Poynting's vector.

Although it might be claimed that particles of a finite mass need not obey the classical wave formulas for the flux and density, as given by Eqs. (1) and (3), four points need to be stressed:

(1) No experiment has ever revealed any wave flux and energy densities (or particle flux and particle densities) not in agreement with Eqs. (3) and (1). Electron diffraction patterns do not appear to be any different from the patterns produced by light. No distinction between the particle flux and density of particles of a finite mass and photons has ever been observed. And, in fact, no one has ever even seriously proposed that such a difference should exist.

(2) The energy flux and density of sound waves is given precisely by Eqs. (3). When a sound wave is quantized and is represented as a stream of particles, a stream of phonons, it must still obey Eqs. (3) and (1). Phonons moving at a velocity less than that of light can be represented as having each a finite mass. The classical theory of sound in conjunction with quantum theory thus requires that Eqs. (1) and (3) be also applicable to particles of a finite mass.

(3) It is only reasonable to require the laws for zero-mass particles to be compatible with the laws for particles of a finite mass when the mass is allowed to go to zero.

(4) Finally, we have no other factual basis at the present which allows us to make any other postulation. Maybe in the future some experiments will reveal a difference between the wave behavior of particles of a finite mass and those of zero mass, but at the moment there is no known reason for making such a distinction.

4. TIME-AVERAGE PARTICLE TRAJECTORIES

The specification of particle trajectories given by Eqs. (2) and (3) is perfectly general, being appropriate for transient as well as for steady-state problems. The classical case of light is, however, generally limited to the case of time-harmonic waves, where only the time-average energy flux is observed. In this case it is convenient to define approximate time-average trajectories as integrals of

$$d\mathbf{r}/dt = \bar{\mathbf{w}} = \langle \mathbf{J} \rangle_t / \langle \rho \rangle_t = \langle \mathbf{S} \rangle_t / \langle \varepsilon \rangle_t \tag{4}$$

It can be shown that these approximate time-average trajectories, represented by Eq. (4), deviate from the true instantaneous trajectories by at most half a wavelength at any particular instant. By macroscopic standards (the usual observational situation) this is an entirely negligible error. Thus, Young's double pinhole experiment can be adequately described using this time-average approximation, Eq. (4), for the particle trajectories.

5. QUANTUM POTENTIAL

A potential that guides the particles along the specified trajectories, the "quantum potential," can be readily defined. For a slow particle with ordinary mass not subject to a classical potential, the kinetic energy is given by $mw^2/2$, where m is the mass of the particle. At infinity away from any boundaries the total energy E may be taken equal to the kinetic energy, where the particle velocity is equal to the phase velocity (see the following section); thus, $E = mu^2/2$. The quantum potential U_Q for a slow-moving particle then becomes

$$U_Q/E = 1 - w^2/u^2 \tag{5}$$

The quantum potential for a fast particle can be similarly obtained using non-Newtonian mechanics. The approximate time-average quantum potential which guides slow particles along the time-average trajectories, as defined by Eq. (4), is defined by

$$\bar{U}_Q/E = 1 - \bar{w}^2/u^2 \tag{6}$$

The quantum potential for photons may also be derived. Conserving the tangential component of the momentum for a photon crossing an interface between two media and in agreement with Maxwell theory, we have

$$E = \mathbf{p} \cdot \mathbf{u} \tag{7}$$

where E is the total energy equal to $\hbar\omega$, $\mathbf{p}$ is the momentum, and $\mathbf{u}$ is the phase velocity assumed to the equal to the photon velocity. In general $\mathbf{u}$ must be replaced by $\mathbf{w}$ in Eq. (7). Using ordinary non-Newtonian mechanics, the kinetic plus rest every is given by

$$E - U = \mathbf{p} \cdot \mathbf{w}c^2/w^2 \tag{8}$$

where U is the potential energy. Combining Eqs. (8) and (7), with $\mathbf{w}$ for $\mathbf{u}$, the quantum potential for photons is given simply by

$$U_Q(\text{photon})/E = 1 - c^2/w^2 \tag{9}$$

Since $c^2/w^2 > 1$, the quantum potential for photons is negative. For example, a photon passing into a material medium falls into a potential well whose depth is $-(n^2 - 1)$, where $n = c/u = c/w$ is the index of refraction (in agreement with Newton's conjectures).

6. THE FREE PARTICLE

Substituting the plane-wave solution to the scalar-wave equation,

$$\Psi = A \sin(kx - \omega t) \tag{10}$$

where k is the propagation constant, ω the angular frequency, and A an arbitrary amplitude constant, into Eqs. (3), one gets

$$\varepsilon = A^2 k^2 \cos^2(kx - \omega t), \qquad S = A^2 k\omega \cos^2(kx - \omega t) \tag{11}$$

The instantaneous free-particle velocity from Eq. (2) [as well as the time-average velocity, Eq. (4)] is then

$$dx/dt = k\omega/k^2 = u \tag{12}$$

The free-particle velocity matches the phase velocity u. This result follows necessarily from the *exact* resolution of the wave-particle problem as specified by Eq. (2). Using other arguments, the author[14] had already shown in 1965 that it was only physically reasonable to choose the phase velocity equal to the particle velocity.

7. YOUNG'S DOUBLE PINHOLE EXAMPLE

The solution to the scalar wave equation for two point sources of equal amplitude and in phase a distance $2D$ apart, placed on the y axis at $+D$ and $-D$, is given by

$$\Psi = A \sin(kR_1 - \omega t)/R_1 + A \sin(kR_2 - \omega t)/R_2 \tag{13}$$

where

$$\mathbf{R}_1 = x\mathbf{i} + (y - D)\mathbf{j}, \qquad \mathbf{R}_2 = x\mathbf{i} + (y + D)\mathbf{j} \tag{14}$$

where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors in the x and y directions. Solutions off the xy plane can be obtained by a rotation about the y axis. Substituting Eqs. (13) and (14) into Eqs. (3) and taking time averages over a period gives

$$\begin{aligned} \langle \varepsilon \rangle_t &= (A^2/4R_1^2R_2^2)\{R_1^2/R_2^2 + R_2^2/R_1^2 + 2k^2(R_1^2 + R_2^2) + 2k^2R_1R_2 \cos\xi \\ &\quad + 2(\mathbf{R}_1 \cdot \mathbf{R}_2/R_1R_2)[(1 + k^2R_1R_2)\cos\xi + \xi \sin\xi]\} \\ \langle \mathbf{S} \rangle_t &= (\omega A^2/2R_1R_2)\{(\mathbf{R}_1/R_1^2)[kR_2 + kR_1 \cos\xi + \sin\xi] \\ &\quad + (\mathbf{R}_2/R_2^2)[kR_1 + kR_2 \cos\xi - \sin\xi]\} \end{aligned} \tag{15}$$

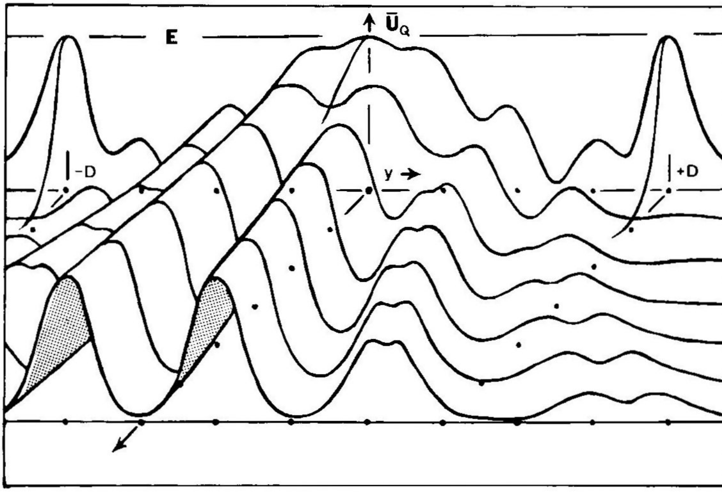

Fig. 2. Time-average potential $\bar{U}_Q$ which yields two-pinhole interference, Eq. (6), (4), and (15), where the distance between pinholes is $2D = 3.6$ times the wavelength. Measure marks are $D/4$ apart.

where

$$\xi = k(R_2 - R_1) \tag{16}$$

Substituting this result, Eq. (15), into Eq. (4) yields the slope of the time-average trajectories $\langle S_y \rangle_t / \langle S_x \rangle_t$ as a function of position. The time-average trajectories for the case $D = 1.8\lambda$, where $\lambda = 2\pi/k$, are shown in Fig. 1. These curves were obtained by indicating the slopes on a rectangular grid and then sketching in the appropriate trajectories by eye. The trajectories were chosen so that they exit isotropically from each of the pinholes. Although a more detailed computerized program of integration would be preferable, Fig. 1 is adequate to indicate the nature of the particle flow.

The time-average quantum potential $\bar{U}_Q$ which guides slow particles along the trajectories yielding the Young's double-pinhole interference, as shown in Fig. 1, may be obtained from Eqs. (6), (4), and (15). The result is shown in Fig. 2. (This result may be compared with the artificial quantum potential obtained by Philippidis, Dewdney, and Hiley[17] for the double-slit case, which does not yield the Poynting's vector or the observed particle flow.)

8. PARTICLE IN A BOX

A standing wave on a string, a standing electromagnetic wave on a wire, or a standing sound wave in an organ pipe are all mathematically equivalent to photons or other particles that exhibit "wave" behavior confined in a one-dimensional box. The wave function Ψ may be assumed to vanish at the boundaries of the box at $x = 0$ and $x = L$. The standing-wave solution may then be taken as

$$\Psi = A \sin kx \sin \omega t \tag{17}$$

where A is the amplitude and k is the propagation constant which is restricted to the eigenvalues

$$k = n\pi/L \tag{18}$$

where n is an integer. The time has been chosen as zero when $\Psi = 0$.

For particles of a finite mass, the case of primary interest here, the propagation constant is given by the de Broglie condition and the frequency ω is chosen to make the classical particle velocity for a free particle v equal to the phase velocity u (as discussed above in Section 6 and in Ref. 14); thus

$$k = p/\hbar \quad \text{and} \quad \omega = pv/\hbar = 2E/\hbar \tag{19}$$

where p is the classical particle momentum. The energy eigenvalues from Eq. (18) and the first of Eqs. (19) are given by

$$E = p^2/2m = n^2\pi^2\hbar^2/2mL^2 \tag{20}$$

(in agreement with the traditional quantum theory).

From Eqs. (3) and (17) the energy density and flux become

$$\begin{aligned} \varepsilon &= k^2A^2(\cos^2 kx \sin^2 \omega t + \sin^2 kx \cos^2 \omega t)/2 \\ S &= -k\omega A^2 \cos kx \sin kx \sin \omega t \cos \omega t \end{aligned} \tag{21}$$

The instantaneous particle velocity from Eq. (2) and (20) is then

$$w = dx/dt = -u \sin 2kx \sin 2\omega t/(1 - \cos 2kx \cos 2\omega t) \tag{22}$$

where $u = \omega/k$ is the phase velocity. This result is immediately integrable. The position as a function of time is then given by

$$2k(x - x_0) = \sin 2kx \cos 2\omega t \tag{23}$$

where the position $x = x_0$ represents a mean position which occurs when $2\omega t = (2m' + 1)\pi/2$, where m' is an integer.

Since $\cos 2\omega t$ is restricted to values between -1 and $+1$, the values of x are restricted by the condition

$$-1 \leqslant 2k(x - x_0)/\sin 2kx \leqslant 1 \tag{24}$$

This means that the turning points of the motion, x_1 and x_2, where $x_1 \leqslant x_0 \leqslant x_2$, are given by

$$\sin 2kx_1 = 2k(x_0 - x_1) \quad \text{and} \quad \sin 2kx_2 = 2k(x_2 - x_0) \tag{25}$$

The position x as a function of the time t, Eq. (23), may be readily computed. The result is shown in Fig. 3. A uniform distribution of mean positions x_0 has been chosen in order to display the behavior of $\rho = \kappa\varepsilon$, where ε given by the first of Eqs. (21). Any distribution of the mean position x_0 would appear to be admissible here; but in order to agree with what is observed in nature, the particle density must be chosen as $\rho = \kappa\varepsilon$, where ε is given by the first of Eqs. (21).

It may be noted from Eq. (23) or Fig. 3 that the particle in a box is confined to only a portion of the box. It is confined to one of n possible cells (or more precisely to one of $2n$ possible half-cells). Since energy cannot flow across a node (cf. the nodes on a vibrating string), this cellular motion was only to be expected.

It might at first appear unreasonable that photons in a cavity, for example, are not free to travel back and forth across the entire width of the cavity. There is, however, evidence that they are confined to cells. First, quantum statistics for photons does not permit the interchange of two photons to be counted as another microstate. The physical reason is now clear: The photons are frozen into cells and cannot physically interchange their positions, as is possible in Boltzmann statistics. (It might also be

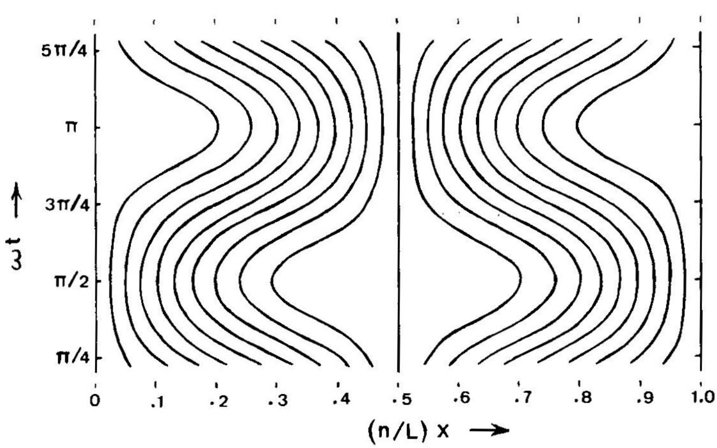

Fig. 3. Particle trajectories as a function of time for a particle in a box for a uniform distribution of mean positions x_0, Eq. (23).

mentioned that the bound state is suppose to be entirely motionless in the traditional and de Broglie–Bohm quantum theories, which would also preclude the possibility of photons being able to physically interchange their roles.)

Second, there is further evidence that the photon in a cavity actually find themselves in a "solid-state" array. Light from a cavity, even at elevated temperatures, is found to be coherent when the light admitted through a small hole is divided by a semitransparent mirror into two beams which are allowed to interfere with each other. The light from a macroscopic region of the cavity can produce essentially zero interference minima. The photons in the cavity must be coupled together in a regular array to produce such complete coherence. Wide-angle interference experiments, even to 180°, also demonstrate the coherence of a light source. When the interfering beams are allowed to have a path difference of about a meter, the coherence is still observable. This means that the cavity acts as a laser for times of the order of 10^{-9} sec. This time may seem very short in comparison to everyday experience; but it is extremely long when compared to the period of oscillation of visible light of about 10^{-15} sec. The entire photon array remains fixed in space for about 10^6 oscillations. Since the coherence for very short path differences is never lost, it means that the whole rigid array of photons drifts slowly in time through space. The cellular restriction and motion described here (which also follows from the *exact* resolution of the classical wave-particle problem) should, therefore, be regarded as a physically real possibility.

A feature that may appear unexpected is the fact that the period of the particle motion is one-half of the period of the associated "wave." The energy on a vibrating string flows back and forth for each half-cycle of the vibrating string.

It is important to note that the motion prescribed by Eq. (23) and shown in Fig. 3 is completely classical, or Newtonian, in character. The position x_0' and velocity w_0' at the time $t = 0$ (or more appropriately for the constant of integration as chosen here, the position x_0 and velocity $w_0 = -u \sin 2kx_0$ when $t = \pi/4\omega$) determine uniquely the subsequent motion, giving the discrete position of a point particle at any subsequent instant. No time-average prediction is involved here. (The present example illustrates the fact that the present theory goes far beyond the traditional or de Broglie–Bohm quantum theories.)

The quantum potential U_Q, Eq. (5), may be found explicitly for the present example. It should be remarked that the quantum potential is a "fictitious potential." It contains no information not contained in the particle motion. It merely offers an alternate method for representing the particle motion. From Eqs. (5) and (22), using (23) to eliminate t, we obtain

$$U_Q/E = 1 - w^2/u^2 = 1 - \sin^2 2kx[\sin^2 2kx - 4k^2(x-x_0)^2]/[\sin 2kx - 2k(x-x_0)\cos 2kx]^2 \quad (26)$$

This result, Eq. (26), is shown in Fig. 4 for various choices of x_0. The quantum potential is zero for $x = x_0$. It equals E at the turning points x_1 and x_2, given by Eqs. (25). For the particular choices $x_0 = 0$, $L/2n$, or L/n, stagnation occurs. Once a particle is placed in these positions with zero velocity, it remains fixed at these positions for all time. (This situation is then reminiscent of the Broglie–Bohm theory where *all* bound particles are motionless.)

9. DISCUSSION

Quantum theory cannot be limited to the submicroscopic domain. Light and sound, for example, preserve their photon and phonon character even on

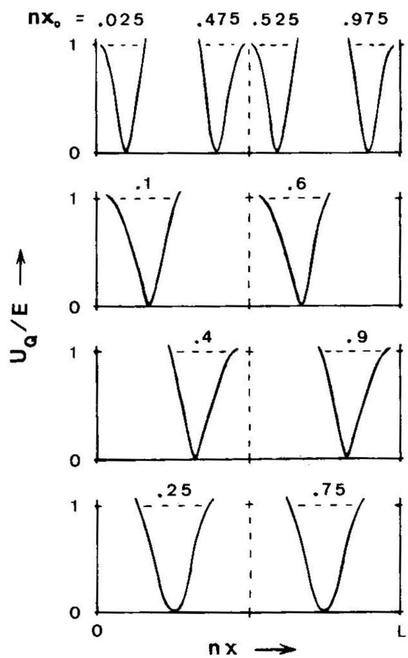

Fig. 4. The quantum potential for a particle in a box, Eq. (26), for various choices of the constant of integration x_0.

a macroscopic scale. An adequate quantum theory must, therefore, not only describe submicroscopic phenomena but must also describe in a self-consistent fashion the relevant macroscopic observations. In particular, an adequate quantum theory must show precisely how photons, phonons, and other quantum particles yield the observed macroscopic "wave" phenomena. Laboratory observations of macroscopic "wave" phenomena are summarized in the highly successful wave theory. An adequate quantum theory must, therefore, necessarily yield the identical mathematics of the classical wave theory when extended to the relevant macroscopic phenomena in order to yield the predictions of the classical wave theory.

The macroscopic "wave" observations, or classical wave theory, consequently presents valuable information as to some of the necessary requirements of any adequate quantum theory, no matter whether it is applied submicroscopically or macroscopically. For consistency with classical wave theory and the resolution of the classical wave-particle problem presented here, it may be concluded that an adequate quantum theory should have the following properties:

(1) The Ψ function should be real.
(2) The phase velocity should equal the classical particle velocity.
(3) The particle density should be proportional to the energy density, the first of Eqs. (3). (The traditional claim that $\Psi\Psi^*$ represents the particle density is seen to be in error, even for the time-average case.)
(4) The particle flux density should be proportional to the energy flux, the second of Eqs. (3). (Again the traditional theory is seen to be in error.)
(5) Time variations must be chosen to be compatible with classical transient "wave" phenomena. (The traditional theory which always presupposes a simple time-harmonic time variation is inadequate to handle transient phenomena.)

Philippidis, Dewdney, and Hiley[17] assumed that the traditional quantum theory is essentially correct. They then used an *ad hoc* procedure for generating discrete particle trajectories in an attempt to derive the classical "wave" result. They did not obtain the observed result. The spirit of the present investigation is turned around: The known classical "wave" observations are used to derive necessary conditions that an adequate quantum theory should satisfy.

An extension of the present theory to the "wave" behavior of particles in a classical force field is straightforward; but the present paper, being limited primarily to the classical wave-particle problem, carries the matter no further. (For further developments see Wesley[20].)

Since the "wave" behavior of particles can be assigned to the quantum potential U_Q, it can be speculated that U_Q and quantum behavior in general involving $\hbar$ is a consequence of a standing electromagnetic field that couples together in phase the electrons in the atoms in macroscopic boundaries. In this way the quantum potential could become imprinted on the boundaries.

Although the classical wave-particle problem is resolved here by showing how point particle motion can yield "wave" behavior exactly, it does not really allow one to say that particles are actually involved. The underlying reality could be either a true wave in a material medium, or a flux of particles; there is no way to decide from the "wave" behavior alone. A more subtle wave-particle problem, therefore, remains unresolved. It can be hoped, however, that processes of creation and absorption of particles (such as the photoelectric effect) might be able to resolve the matter.

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