FARADAY'S LAW IS WRONG; CHANGING MAGNETIC FLUX DOES NOT CAUSE INDUCTION

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ABSTRACT

The force of induction $\mathbf{F}_i$ on a charge q is caused by the time rate of change of the vector magnetic potential **A** (produced by a closed current loop source) at the charge; thus, $\mathbf{F}_i = - q d\mathbf{A}/dtc$. The time rate of change of the magnetic flux $d\Phi/dt$ is also caused by $d\mathbf{A}/dt$. Thus, the Faraday law, $emf = -\partial\Phi/\partial tc$, merely relates two effects without any cause being specified. The difficulties and failures of Faraday's law resulting from its usual erroneous interpretation as a cause and effect relationship are discussed. A brief introduction to the general theory of induction is presented.

1. INTRODUCTION

The proper law of induction (for an **A** field due to current loops) states: The force of induction $\mathbf{F}_i$ on a charge q (the *effect*) is produced by the negative time rate of change of the magnetic vector potential **A** at the charge (the cause); thus,

$$\mathbf{F}_i/q = - d\mathbf{A}/dtc . \tag{1}$$

Faraday's law of electromagnetic induction refers to the very special case of an entire closed circuit and the net potential or emf (electromotive force) induced around the entire closed circuit,

$$emf = \oint ds \cdot \mathbf{F}_i/q . \tag{2}$$

Faraday's law,

$$emf = -\partial\Phi/\partial tc , \tag{3}$$

is usually interpreted (incorrectly) as meaning the change in magnetic flux through the closed circuit, $\partial\Phi/\partial tc$, is the cause producing the emf, the *effect*. But the two sides of Eq.(3) are merely two *effects* produced by the common cause, the time changing magnetic vector potential, $\partial\mathbf{A}/\partial t$. The right side of Eq.(3) yields

$$\partial\Phi/\partial t = \int da\, \mathbf{n} \cdot \partial\mathbf{B}/\partial t = \int da\, \mathbf{n} \cdot (\nabla \times \partial\mathbf{A}/\partial t) = \oint ds \cdot \partial\mathbf{A}/\partial t, \tag{4}$$

where n is the unit normal to the surface bounded by the circuit, $\mathbf{B} = \nabla \times \mathbf{A}$, and Stokes's theorem yields the line integral around the loop. And the left side of Eq.(3) using Eqs.(2) and (1) yields

$$emf = \oint ds \cdot \mathbf{F}_i/q = - \oint ds \cdot \partial\mathbf{A}/\partial tc . \tag{5}$$

Thus, both sides of Eq.(3) are produced by precisely the same cause.

The force of induction, Eq.(1),. (for **A** due to current loops) is sometimes recognized in textbooks teaching the traditional Faraday-Maxwell theory (e.g., Symthe 1950); but its significance is conveniently ignored or overlooked.

2. THE FARADAY LAW DOES NOT SPECIFY THE FORCE OF INDUCTION ON A CHARGE AT A POINT

From the definition of the emf, Eq.(2), it is obvious that the cause of the emf has to be a force of induction F_i, that forces the charge q around the circuit. Since the Faraday law, Eq.(3), does not specifiy this force explicitly; it is not a proper physical law.

Since the Faraday law specifies the resultant net emf around an entire closed circuit and, thus, the integral average of the force of induction around the,circuit; it cannot specify the force of induction acting on an individual charge at a point. It is impossible to deduce the integrand from the value of the integral.

A proper physical law should provide the essential details and not merely integral averages. The law of induction for A fields due to closed current loops, Eq.(1), specifies the force acting at a point on an individual charge. It is a proper physical law in contrast to the Faraday law.

The Faraday law, being restricted to the voltage induced in an entire closed circuit, fails for more general situations. The proper law of induction, Eq,(1), can predict induction in general, such as the voltage induced between the ends of a straight piece of wire.

3. A MAGNETIC B FIELD IS NOT NECESSARY TO PRODUCE INDUCTION

Faraday's law, Eq.(3), is usually interpreted to mean that a time-changing magnetic B flux through a circuit is *necessary* to produce, or cause. induction. But according to the proper law of induction given by Eq.(1) this is not true. Only a time-changing magnetic potential A field is necessary; and whether or not $B = \nabla \times \Delta$ happens to be zero or not zero is a matter of indifference.

Induction without a B field is readily demonstrated. the emf induced in the outer secondary winding of a toroidal transformer occurs in a region where the B field, being contained within the inner toroidal winding, is always precisely zero. Further examples of induction in regions where the magnetic B field is zero are provided by the Hooper (1974) - Monstein (1992) experiment, the Aharonov Bohm effect (Wesley 1998a), and the Marinov motor (Wesley 1998b). In all of these cases the necessary time-changing A field is not, of course, zero.

It should be noted that the magnetic B field is not a fundamental quantity. It is merely a particular property of the A field, namely, the curl of A, or $B\ \nabla \times A$. The A field, being defined directly in terms of the current source J(r,t), is fundamental; thus,

$$\mathbf{A}(\mathbf{r},t) = \int d^3r'\, \mathbf{J}(\mathbf{r}', t)/|\mathbf{r} - \mathbf{r}'| . \tag{6}$$

Thus the proper law of induction Eq.(1), valid for closed current loop sources for the A field, involving the **A** field is more fundamental than the Faraday law involving the **B** field flux over an area.

4. THE FARADAY LAW CLAIMS AN EFFECT INDEPENDENT OF THE CAUSE

According to the Faraday law, as usually interpreted, the timechanging magnetic **B** field in the empty area inside a wire circuit is supposed to be the *cause* of the emf induced in the wire circuit itself. And the time-changing **B** field need not be in the wire circuit :itself. Thus, like astrology, the Faraday law offers no physical mechanism coupling the spacially separated *cause* (the time-changing **B** field inside the area of the circuit) with the produced *effect* (the emf in the wire itself). As may be seen from the proper law of induction, Eq. (1) the **B** field in the area enclosed by the circuit has absolutely nothing to do with the force of induction in the wire itself, that produces the observed emf.

5. INDUCTION NEED NOT INVOLVE A CLOSED CIRCUIT

After a switch is opened in a wire circuit, that was originally experiencing an induced emf, the area originally defined by the closed-circuit no longer exists. According to Faraday's law the magnetic flux can thus no longer exist; and induction should no longer occur. Yet, according to the proper law of induction, Eq.(1), the same force of induction $\mathbf{F}_i$ continues to exist in the wire. A charge separation becomes established in the wire producing an electrostatic counter force that results in

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a zero net force on the charges in the wire. An electrometer placed across the open switch will continue to measure precisely the same induced voltage or emf as before. The phenomenon of induction remains the same with switch open or closed.

The gap in the circuit represented by the open switch can be topologically opened so wide that the open circuit becomes simply a straight piece of wire. According to the proper law of induction, Eq.(1), the force of induction can produce a potential difference between the ends of a straight piece of wire, such as measured by Kennard (1917).

Topologically the Faraday law fails, when a closed circuit and thus a magnetic flux through the circuit cannot be defined. To try to rescue the Faraday law (as well as the Maxwell theory) it is sometimes claimed that closed circuits always exist; because physical gaps in open circuits can always be bridged with imaginary mathematical curves through empty space. Unfortunately, such imaginary curves cannot be uniquely chosen; and they can yield any arbitary (within limits) value for the magnetic flux Φ through the imaginary area generated. This ploy cannot rescue Faraday's law.
The proper law of induction, Eq.(1), gives the force on an individual charge, such as the force on an individual electron in the Aharonov-Bohm experiment (Wesley, 1998a), which is clearly impossible for the Faraday law.

6. FARADAY'S LAW DOES NOT YIELD LOCALIZED INDUCTION

The Faraday law, Eq.(3), does not say where in the loop the actual emf occurs. It is generally assumed that Faraday's law implies an emf uniformly generated around the closed circuit. But Francisco Muller (1987) performed some ingeniously simple experiments to demonstrate the fact that the net induced emf in the entire circuit can be generated in some portions and not in other portions of the circuit. Faraday's law, thus, fails; because it cannot say where in the loop the emf is generated. In contrast, the proper law of induction, Eq.(1), readily accounts for the observed localized induction.

7. THE GENERAL THEORY OF INDUCTION

The proper law of induction, Eq.(1), is only valid for the **A** field produced by closed current loop sources. The theory of induction needs to be now generalized to include any general current source.

Since induction involves slowly varying effects in terms of v/c; the original Weber (1848) electrodynamics is appropriate, where the force on a charge q at **r** with velocity **v** and acceleration **a** due to a charge q' at **r'** with velocity **v'** and acceleration **a'** is given by

$$\mathbf{F}(\text{Weber}) = qq'(\mathbf{R}/R^3)\,[1 + V^2/c^2 - 3(\mathbf{V}\bullet\mathbf{R})^2/2C^2R^2 + \mathbf{R}\bullet\mathcal{A}/c^2] \tag{7}$$

where

$$\mathbf{R} = \mathbf{r} - \mathbf{r}',\ \mathbf{V} = \mathbf{v} - \mathbf{v}',\ \mathcal{A} = \mathbf{a} - \mathbf{a}'. \tag{8}$$

The force of induction arises from the relative acceleration between the two charges, given by the last term in the brackets in Eq.(7). Weber himself (1848) showed that his theory yields the Faraday law.

When extended to fields produced by an extended source of many charges q', as in the Weber-Wesley (1990, 1997) field theory, the general force of induction $\mathbf{F}_i$ becomes

$$c\mathbf{F}_i/q = \mathbf{a}\Phi/c - (\mathbf{a}\bullet\nabla)\mathbf{G}/c - d\mathbf{U}/dt, \tag{9}$$

where Φ is the usual electrostatic potential and **G** and **U** are potential fields defined by

$$\mathbf{G}(\mathbf{r}) = \int d^3\mathbf{r}'\mathbf{R}\,\rho'(\mathbf{r}')/R, \qquad \mathbf{U}(\mathbf{r}) = \int d^3\mathbf{r}'\mathbf{R}\,[\mathbf{R}\bullet\mathbf{J}'(\mathbf{r}')]/R^3c, \tag{10}$$

where $\rho(\mathbf{r}')$ is the charge density and $\mathbf{J}'(\mathbf{r}')$ is the current density.

The terms on the right of Eq.(9) involving the acceleration a of the detector charge q represent Lenz' law for the induced back emf. And, when applied to gravitation, $-$Gmm' replacing qq', these terms yield Mach's principle for the inertial reaction force as the product of the gravitation potential at the mass m acting on the mass m (Wesley 1991).

The vector **U** potential, the second of Eqs.(10), may be regarded as a generalization of the magnetic vector **A** potential to include all possible current sources in addition to closed current loops. Since the Weber theory is a relativity theory that involves only the separation distance R = I**r** - **r'** I between q and q' and its time derivatives; only the relative velocity component along the line joining the two charges can be involved. Thus, the current source of the potential U field must be the **R** component of the current density **J'(r')**, or **R•J'(r')**/R. In addition, the force must act along **R**; so the source of the potential **U** field becomes $\mathbf{R}(\mathbf{R}\bullet\mathbf{J}')/R^2$, as indicated in the second of Eqs.(10), instead of simply **J**, the source of the **A** field.

It may be noted that U may be written as

$$\mathbf{U} = \mathbf{A} - \nabla\Gamma. \tag{11}$$

where

$$\Gamma = \int d^3 r'\,\mathbf{R}\bullet\mathbf{J}'(\mathbf{r}')/Rc. \tag{12}$$

For the case of closed current loop sources, the case of greatest practical interest, $\Gamma = 0$. In particular for a particular tube of closed current loop flow

$$c\Gamma = i\int ds.\mathbf{R}/R = i\int \delta_s R = iR\,|_0^0 = 0, \tag{13}$$

where i is the current in the tube.

To further stress the fact that **U** is a generalization of the **A** field, it may be noted that the magnetic **B** field is given by

$$\mathbf{B} = \nabla \times \mathbf{U} = \nabla \times \mathbf{A}, \tag{14}$$

for any arbitrary current source.

The total time derivative of the U field, appearing in Eq.(9), may be expanded to read

$$d\mathbf{U}/dt = \partial\mathbf{U}/\partial t + (\mathbf{V}\bullet\nabla)\mathbf{U} + (\mathbf{U}\bullet\nabla)\mathbf{V}, \tag{15}$$

where **V** is the relative velocity between the charge q and the rigid source of the **U** field. This result (15) may be regarded as a theorem valid for any vector field, as proved by Wesley (1999). The first term on the right of Eq.(15) gives the time change of U itself. The second term on the right gives the apparent time change of the U field due to the motion of the charge through a space-changing U field in the direction of U. The last term on the right gives the apparent time change of the U field due to the charge changing its direction of motion.

The greatest practical interest is for no net source charges, $\rho'(\mathbf{r}') = 0$, and for closed current loop sources; thus,

$$-c\mathbf{F}_i/q - d\mathbf{A}/dt = \partial\mathbf{A}/\partial t + (\mathbf{V}\bullet\nabla)\mathbf{A} + (\mathbf{A}\bullet\nabla)\mathbf{V}. \tag{16}$$

This result (16) accounts for unipolar induction: since the terms for notional induction may be written as

$$-(\mathbf{V}\bullet\nabla)\mathbf{A} - (\mathbf{A}\bullet\nabla)\mathbf{V} - \mathbf{V}\times(\nabla\times\mathbf{A}) + \mathbf{A}\times(\nabla\times\mathbf{V}) - \nabla(\mathbf{V}\bullet\mathbf{A}), \tag{17}$$

where the only non-vanishing term of interest is $\mathbf{V}\times(\nabla\times\mathbf{A})$. The second term on the right of Eq. (16) accounts for the Aharonov Doha effect (Wesley 1998a). the Hooper (1974) - Monstein (1992) experiment, and the force to drive Marinov's motor (Wesley 1998b, Phipps 1998).

It may be noted that the Weber-Wesley (1990, 1997) field theory, being valid in absolute space, is not a relativity theory. It is not immediately compatible with the theory of induction presented here. It is not clear what approximation of the absolute space theory is needed to derive the relativity induction theory.

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