Failure of the Uncertainty Principle

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Abstract
The uncertainty principle is shown to be simply a condition for the geometrical optics approximation to be valid, so it fails when wave effects are involved. Using ordinary scientific evidence, the uncertainty principle is shown to fail by orders of magnitude for six explicit empirical examples: a light microscope, the hydrogen atom, a pocket radio, a scanning light microscope, beta decay, and Wiener fringes. The alternative causal quantum theory that yields precise wave effects without any uncertainties is discussed.

Key words: uncertainty principle, quantum theory

1. BACKGROUND

The uncertainty principle is usually stated in terms of the measurement process, but is generally interpreted to mean a limit on the knowledge we can have about the state of a system. The hypothetical measurement process that is considered is artificially restricted and contrived to make it appear that the knowledge we can have about a state of a system is far less than the knowledge that we actually have from actual observations.

In particular, it is traditionally claimed that to determine the simultaneous values of two canonically conjugate variables, p and q, for a single particle it is necessary to measure the variables p and q simultaneously. After determining p and q for a single particle, the uncertainty principle then says that one will find that the uncertainties in p and q will satisfy the relation $\Delta p \Delta q > \hbar$. But this representation of the uncertainty principle is unsound both logically and scientifically for the following reasons.

First, "uncertainties," or experimental errors of actual scientific measurements, are defined statistically as the standard deviation from the mean of many different measurements on many different particles. It is impossible to define and to determine a Δp or a Δq if only one single value of p and one single value of q are measured and known for a single particle. The intrinsic "uncertainties" of the uncertainty principle that are to be associated with a single observation of a single particle are, thus, impossible to define operationally or scientifically in the laboratory.

Second, it is not at all true that one must measure p and q of a single particle simultaneously in order to determine the simultaneous values of p and q for a single particle. Science employs a host of strategies to gain knowledge about the state of a particular system or particle at a given instant without measuring all the variables of interest simultaneously on only a single system or particle. For example, a stream of many particles all prepared in the same state can be sampled. The position q of some particles can be measured from time to time, while the momentum p of other particles can be measured at other times. The simultaneous values of p and q for any particular particle in the stream can then be deduced and known. One need not dissect a living cat to investigate the simultaneous structure and functioning of a cat's heart.

Third, even if one were to actually measure p and q simultaneously, there is absolutely no empirical evidence available that supports the claim that the observed errors will satisfy the uncertainty relation $\Delta p \Delta q > \hbar$.

Fourth, in order for any principle to have scientific meaning, it must tell us something about nature. It must tell us something about the state of a system or particle. If the uncertainty principle is interpreted as referring solely to the difficulties of making certain unusual restricted measurements, then the uncertainty principle can tell us nothing about nature itself.

In actual practice the uncertainty principle is rarely interpreted as referring simply to the measuring process alone. In actual practice, as usually applied, the uncertainty principle is generally interpreted to mean a limit on the knowledge we can have about the state of a system. This interpretation concerns nature itself and is of scientific interest. The question is then, Is our knowledge about physical states in nature in fact limited by the uncertainty relation $\Delta p \Delta q > \hbar$?

2. THE UNCERTAINTY PRINCIPLE AS A CONDITION FOR THE GEOMETRICAL OPTICS APPROXIMATION

A source of coherent photons or other quantum particles, when separated into two beams and recombined with a phase difference

$$\Delta\phi = k\Delta q, \tag{1}$$

where k is the propagation constant, can exhibit no interference when the path difference Δq is large enough. This inability to show interference is apparently produced by a spread of wavelengths, or of the propagation constants, radiated by the original source; thus

$$\Delta k = 2\pi\Delta\lambda/\lambda^2, \tag{2}$$

where $\Delta\lambda$ is the "line width." This produces a corresponding spread in the phase difference given by Eq. (1); thus

$$\delta(\Delta\phi) = \Delta k\Delta q = 2\pi(\Delta\lambda/\lambda^2)\Delta q. \tag{3}$$

It is clear that the relative coherence between the two beams will be destroyed when the spread in phase differences becomes greater than 2π; thus when

$$\Delta k\Delta q = 2\pi(\Delta\lambda/\lambda^2)\Delta q \geq 2\pi, \tag{4}$$

the equality defines the "coherence length" for Δq. The de Broglie wavelength condition $\lambda = h/p$ for a plane wave then gives

$$\Delta p\Delta q \geq h. \tag{5}$$

Similar considerations involving the frequency and the Planck frequency condition yield for the particle energy E

$$\Delta E\Delta t \geq h. \tag{6}$$

These results (5) and (6) are the Heisenberg[1] uncertainty principle as originally stated. When h is replaced by $\hbar = h/2\pi$, as is now usually done, the derivation above is not altered, because coherence is also destroyed when the spread in phase differences, Eq. (3) and (4), becomes larger than one radian.

Equation (5) is a condition for the failure of coherence when interference cannot be observed, and the wavelength can no longer play a role. The uncertainty principle is thus simply a condition for the geometrical optics approximation to be valid. Schrödinger[2] based his original quantum theory upon the geometrical optics approximation of Hamilton. Schrödinger's[3] "wave packet," which was supposed to represent a single smeared-out particle, was also contrived to fit the geometrical optics approximation, where wave behavior was not involved and the individual component de Broglie waves were not supposed to be observable. The uncertainty principle, as a condition for the geometrical optics approximation to be valid, is thus seen to be completely compatible with these original ideas of Schrödinger.

Recently Marquardt and Galeczki[4] pointed out that the concept of a "wave packet" and Planck's quantum hypothesis $E_0 = h\nu_0$ are mutually exclusive, since the energies of the Fourier components with frequencies centered at ν_0 do not necessarily sum up to E_0. In any case, the idea that every photon is made up of an infinity of photons destroys the original meaning of the energy quantum.

Quantum particles do interfere, and they do exhibit wave effects involving the de Broglie wavelength explicitly, and these effects are observed without any uncertainties. The geometrical optics approximation of Schrödinger and Heisenberg is thus far, far too crude to describe these precise wave effects. The uncertainty principle can thus be expected to fail by many orders of magnitude for situations involving wave behavior, such as prescribed by physical optics.

3. LIGHT MICROSCOPE

The photons that reveal the details of the interior of a living cell of 10^{-3} cm in diameter are at some time localized within the cell, so the uncertainty in the position of each and every one of these individual photons while in the cell is such that

$$\Delta q_i < 10^{-3}\ \text{cm}, \tag{7}$$

where $\Delta q_i = \Delta x$, Δy, or Δz. If the light is from an ordinary line source with a wavelength of $\lambda = 5 \times 10^{-5}$ cm and with a fractional line width of

$$\Delta\lambda/\lambda < 10^{-6}, \tag{8}$$

then from the de Broglie wavelength condition $p/\hbar = 2\pi/\lambda$, the spread or uncertainty in the magnitude of the momentum is

$$\Delta p/\hbar = 2\pi\Delta\lambda/\lambda^2 < 10^{-1}\ \text{cm}^{-1}. \tag{9}$$

Since $\Delta p \geq \Delta p_x$, Δp_y, or Δp_z, Eqs. (7) and (9) yield for each canonically conjugate pair $\Delta p_x\Delta x$, $\Delta p_y\Delta y$, and $\Delta p_z\Delta z$ the result

$$\Delta p_i\Delta q_i/\hbar < 10^{-4}; \tag{10}$$

so the uncertainty principle, Eq. (5), fails by at least four orders of magnitude for this particular empirical example.

There is no reason to consider any hypothetical "thought experiment" that can be contrived to satisfy the uncertainty principle by arbitrarily requiring a simultaneous measurement of position and momentum of a single photon.[5-7] Ordinary valid scientific evidence gained from *actual* experiments is sufficient to deduce the perfectly reasonable result (10) for each individual photon used to observe the interior of the living cell.

4. HYDROGEN ATOM

Just how exact is the simultaneous position and momentum of the electron in an unexcited hydrogen atom? Since it is known from much scientific evidence that the electron is bound in the hydrogen atom, the uncertainty in the position of the electron must certainly be less than the size of the hydrogen atom itself, or twice the Bohr radius, $a_0 = 5 \times 10^{-9}$ cm; thus

$$\Delta r = \Delta q < 2a_0 = 10^{-8}\ \text{cm}. \tag{11}$$

The uncertainty in the momentum of the electron in the hydrogen atom may be estimated from the uncertainty in the energy levels as evidenced by line widths of the transitions between levels, where it is empirically found that, as above in Eq. (8),

$$\Delta\lambda/\lambda = \Delta E/E = 10^{-6}. \tag{12}$$

Assuming this entire uncertainty can be ascribed to the uncertainty in the kinetic energy of the electron in the ground state, $E = p^2/2m$, the fractional uncertainty in the linear momentum of the electron in the radial direction p_r is such that

$$\Delta p_r/p \leq \Delta p/p = \Delta E/2E = 5 \times 10^{-7}. \qquad (13)$$

Since the angular momentum $a_0 p$ in the ground state is quantized as $\hbar$, Eq. (13) yields the uncertainty in the radial momentum as

$$\Delta p_r/\hbar \leq \Delta p/\hbar = 10^2 \text{ cm}^{-1}. \qquad (14)$$

From Eqs. (11) and (14) the uncertainties in radial position and momentum of the electron in the hydrogen atom satisfy

$$\Delta p_r \Delta r/\hbar \leq \Delta p \Delta q/\hbar = 10^{-6} \ll 1. \qquad (15)$$

Since there is only one single electron in the hydrogen atom and since it must have simultaneously both a position and a momentum, result (15) says that the simultaneous uncertainties in position and momentum are known to a precision six orders of magnitude greater than that prescribed by the uncertainty principle, Eq. (5).

It is also of interest to consider the canonically conjugate angular position of ϕ and the angular momentum p_ϕ. Certainly the angular position of the electron is localized within 2π rad, so the angular uncertainty is

$$\Delta\phi < 2\pi. \qquad (16)$$

From Eq. (14) the uncertainty in the angular momentum, $\Delta p_\phi = a_0 \Delta p$, is given by

$$\Delta p_\phi/\hbar = 5 \times 10^{-7}. \qquad (17)$$

Combining Eqs. (16) and (17), the known simultaneous uncertainties in angular position and angular momentum are such that

$$\Delta p_\phi \Delta\phi/\hbar < 3 \times 10^{-6} \ll 1, \qquad (18)$$

where again the uncertainty principle, Eq. (5), is seen to fail empirically by six orders of magnitude.

Since results (15) and (18) are known from the ordinary indirect scientific evidence provided by numerous actual experimental observations, there is no justifiable reason to consider any arbitrary hypothetical thought experiment involving the scattering of electrons or gamma rays from a hydrogen atom that is contrived to produce large enough uncertainties to satisfy the uncertainty principle.[5-7]

5. POCKET RADIO

A pocket radio, 10 cm in its longest dimension, absorbs electromagnetic radiation. Radio waves, being electromagnetic waves, like light, must also exhibit photon behavior. The uncertainty in the localization of the photons absorbed by the radio is no greater than the radio's largest dimension, so

$$\Delta q = 10 \text{ cm}. \qquad (19)$$

If the tuning circuitry is not very precise, the fractional error in the frequency distinguished by the radio might be only

$$\Delta\nu/\nu = \Delta\lambda/\lambda = 10^{-2}. \qquad (20)$$

When the radio is tuned to 1000 kilocycles, the wavelength is $\lambda = c/\nu = 3 \times 10^4$ cm. From the de Broglie wavelength condition $p/\hbar = 2\pi/\lambda$, the uncertainty in the magnitude of the photon momentum is then

$$\Delta p/\hbar = 2\pi\Delta\lambda/\lambda^2 = 2 \times 10^{-7} \text{ cm}^{-1}. \qquad (21)$$

Since an individual photon must have simultaneously both a position and a momentum, the simultaneous uncertainties in *position and momentum for each individual radio photon* absorbed from Eqs. (19) and (21) for any particular Cartesian direction x satisfy the result

$$\Delta p_x \Delta x/\hbar \leq \Delta p \Delta q/\hbar = 2 \times 10^6 \ll 1, \qquad (22)$$

which clearly violates the uncertainty principle, Eq. (5), by six orders of magnitude. Here again the results of actual observations cannot be discounted merely on the basis of arbitrary unwarranted thought experiments.

It might be protested that result (22) does not refer to a single photon because the radio does not respond to a single photon and thus the uncertainty principle is not violated. While it is clearly true that many photons must be absorbed for the radio to respond, result (22) represents a relationship between the position and momentum of each individual photon absorbed quite independent of the flux or number of photons that must be absorbed.

6. SCANNING LIGHT MICROSCOPE

A scanning light microscope with a hole at the end of the pointed probe one-tenth of the wavelength of the light used locates an individual photon when passing through the tip to within the lateral uncertainty of

$$\Delta x = \Delta q = \lambda/10. \qquad (23)$$

Assuming a monochromatic line source for the light used, the fractional uncertainty in the wavelength due to the line width, as mentioned above, Eq. (8), is ordinarily about 10^{-6}. From the de Broglie wavelength condition the uncertainty in the momentum is then given by

$$\Delta p_x/\hbar \leq \Delta p/\hbar = 2\pi\Delta\lambda/\lambda^2 = 2\pi \times 10^{-6}/\lambda. \qquad (24)$$

Combining Eqs. (23) and (8), the simultaneous uncertainties in position and momentum of each and every individual photon passing through the tip satisfies the condition

$$\Delta p_x \Delta x/\hbar \leq \Delta p \Delta q/\hbar = 6 \times 10^{-7} \ll 1. \qquad (25)$$

The uncertainty principle, Eq. (5), is seen to fail in this case by six orders of magnitude. Here again this result (25) is obtained from actual experimental observations, and there is no reason to discount this empirical result on the basis of arbitrarily contrived thought experiments.

In an attempt to justify the uncertainty principle it is sometimes claimed that it refers only to the measuring process and that it is not involved with knowing the properties of unobserved states, which, being unobserved, can never be known anyway. But science is concerned with gaining knowledge about nature herself, whether directly observed or indirectly deduced. A brain surgeon must know, from his anatomy courses, the structure of his patient's brain even before he operates and observes directly his patient's brain for the first time.

7. BETA DECAY

The case of tritium beta decay may be considered. The radius of the tritium nucleus is about[8] 1.7×10^{-13} cm. At the instant the electron leaves the nucleus the uncertainty in its position is then no more than about

$$\Delta r = \Delta q = 3.4 \times 10^{-13}\ \text{cm}. \qquad (26)$$

The energy of the electron emitted is observed to be about $E = 0.02$ MeV. This corresponds to an electron momentum of $p = (2mE)^{1/2} = 8 \times 10^{-18}$ g·cm/s. The simultaneous uncertainty in the momentum of the electron Δp as it leaves the tritium nucleus is most certainly no greater than p itself, so

$$\Delta p_r/\hbar = \Delta p/\hbar < 8 \times 10^{9}\ \text{cm}^{-1}. \qquad (27)$$

The simultaneous uncertainties in position and momentum from Eqs. (26) and (27) then yield

$$\Delta p_r \Delta r/\hbar = \Delta p \Delta q/\hbar < 3 \times 10^{-3} \ll 1, \qquad (28)$$

which violates empirically the uncertainty principle, Eq. (5), by three orders of magnitude.

In the present example nature herself reveals the failure of the uncertainty principle, since no special experiment or measurement is required. The simultaneous uncertainties in the position and momentum of the decay electron, as given by Eqs. (26) and (27), can hardly be disputed even by using some arbitrarily contrived thought experiment.

It might be objected that the quantum theory may, in fact, fail for the atomic nucleus, so this result (28) is then not relevant. But the uncertainty principle is still violated when the electron is a hundred radii from the nucleus.

8. WIENER FRINGES

Since the traditional Copenhagen quantum theory says a single photon is a "wave packet" smeared out over a large number of wavelengths, it is not supposed to be possible to know the precise behavior of a photon within a de Broglie wavelength. Yet everyday long-wave radio transmission and reception involves the detailed behavior of photons within a very small fraction of a wavelength, as already discussed in Sec. 5 above. Wiener,[9] as long ago as 1890, using a very thin photographic film one-thirtieth of a wavelength of the light used, recorded the positions of the loops and nodes of a standing light wave produced by light reflected from a mirror. He thus recorded details of the behavior of light within a de Broglie wavelength. It is, therefore, of interest to see if Wiener's observations also violate the uncertainty principle.

In Wiener's experiment photons are localized within the thickness of the film used, which was $\lambda/30$. For a wavelength of 5×10^{-5} cm the uncertainty in the position of a photon in the film was no greater than

$$\Delta z = \Delta q = 2 \times 10^{-6}\ \text{cm}. \qquad (29)$$

The regularity of the standing wave pattern observed required the uncertainty in the wavelength of the light used to be also less than the thickness of the film,

$$\Delta\lambda < 2 \times 10^{-6}\ \text{cm}. \qquad (30)$$

Using the de Broglie wavelength conditions, the uncertainty in the momentum of the photon in the film is such that

$$\Delta p_z/\hbar \leq \Delta p/\hbar = 2\pi\Delta\lambda/\lambda^2 < 5 \times 10^{3}\ \text{cm}^{-1}. \qquad (31)$$

Combining Eqs. (29) and (31) the simultaneous uncertainties in position and momentum for each photon observed in the film satisfy the condition

$$\Delta p_z \Delta z/\hbar \leq \Delta p \Delta q/\hbar < 10^{-2} \ll 1, \qquad (32)$$

which violates the uncertainty principle, Eq. (5), by two orders of magnitude.

9. THE ALTERNATIVE CAUSAL QUANTUM THEORY

In contrast to the rough geometrical optics approximation, which does not depend upon the wavelength, physical optics yields precise detailed wave behavior that depends explicitly upon the wavelength. Thus a quantum theory that yields precise wave behavior should be based upon physical optics and not upon the geometrical optics approximation of Schrödinger and Heisenberg. The alternative theory[10,11] assumes small quantum particles that can be precisely located within a wavelength, instead of the huge Schrödinger "wave packet" representation of a particle smeared out over hundreds or thousands of wavelengths. This alternative theory is in the spirit of Newton,[12] who believed that photons were very small particles much smaller than half a wavelength (or "fit").

Classically the velocity of energy propagation of a wave that is appropriate for physical optics and sound is defined by the classical Poynting vector $\mathbf{P}$ and the wave energy density E; thus

$$\mathbf{w} = \mathbf{P}/E. \qquad (33)$$

If photons, phonons, and other quantum particles exhibiting wave behavior carry the wave energy and momentum, then their velocity must be precisely the classical wave energy velocity **w**, Eq. (33), in order for a flux of quantum particles to give rise to the empirically observed energy flux **P** and energy density E.

For a scalar wave Ψ the Poynting vector **P** and the energy density E are defined by

$$\mathbf{P} = -\nabla\Psi\partial\Psi/\partial t,$$
$$E = (\nabla\Psi)^2/2 + (\partial\Psi/\partial t)^2/2u^2, \tag{34}$$

where the wave function Ψ has an appropriately defined amplitude constant and an appropriate phase velocity u. The wave function Ψ satisfies an appropriate wave equation. Once Ψ is known as a function of position and time, **P** and E can be obtained by differentiation, Eqs. (34). Then **w** may be found as an explicit function of position and time from Eq. (33). Integrating **w** then yields the motion of a quantum particle along a discrete trajectory as an explicit function of time and the initial conditions. This motion shows how quantum particles yield interference and diffraction patterns without any uncertainties, probabilities, or ambiguities. Knowing the quantum particle velocity **w** from Eq. (33) then permits all observables to be derived precisely in principle.

This alternative causal quantum theory[10] makes the empirically false uncertainty principle superfluous. This alternative theory also obviates many of the other fundamental errors of the traditional Copenhagen quantum theory.

An aspect of light unknown to Newton and Fresnel was its quantization in energy units E first discovered by Planck and prescribed by the Planck frequency condition

$$E = h\nu = \hbar\omega, \tag{35}$$

where $\omega = 2\pi\nu$ is the angular frequency. The propagation constant **k** from Eq. (35) is then quantized in momentum units, given by the de Broglie condition

$$\mathbf{p} = \hbar\mathbf{k}. \tag{36}$$

De Broglie speculated that Eq. (36) should also be true for particles of nonzero rest mass, which was verified by electron diffraction.

Since a classical light wave is empirically defined by a flux of photons, the wave velocity **u** and the photon velocity **v** should be identical[13] for a free-space wave; thus

$$\mathbf{u} = \mathbf{v} = \mathbf{k}\omega/k^2, \tag{37}$$

which must also be true for all quantum particles. From Eq. (36) the angular frequency for all quantum particles is then given by

$$\omega = \mathbf{p}\cdot\mathbf{v}/\hbar. \tag{38}$$

Result (38) is seen to agree with the Planck frequency condition (35) for light. For a slow particle of nonzero rest mass $\omega = 2W/\hbar$, where W is the kinetic energy of the particle. This may be contrasted with the incorrect traditional frequency arbitrarily chosen as $\omega = E/\hbar$, where E is the total particle energy. The empirically correct free space wave for any quantum particle is then the Wesley wave,[13]

$$\Psi = \sin[\mathbf{p}\cdot(\mathbf{r} - \mathbf{v}t)/\hbar]. \tag{39}$$

This empirically correct Wesley wave, Eq. (39), may be contrasted with the physically impossible de Broglie wave, where the frequency is given by $\omega = mc^2\gamma/\hbar$ and the phase velocity, $u = c^2/v$, goes to infinity as the particle comes to rest.

In general, the wave function Ψ is prescribed by the wave equation

$$u^2\nabla^2\Psi - \partial^2\Psi/\partial t^2 = 0, \tag{40}$$

where the phase velocity u is to be taken as the classical particle velocity. For bound systems described by standing waves the wave equation separates into a space part $\psi(\mathbf{r})$ and a time part $T(t)$, where

$$\Psi = \psi(\mathbf{r})T(t), \tag{41}$$

which must then satisfy the two differential equations

$$\nabla^2\psi + k^2\psi = 0, \quad \partial^2T/\partial t^2 + \omega^2T = 0. \tag{42}$$

For slow particles of nonzero rest mass moving in a conservative force field with a potential $V(\mathbf{r})$ the propagation constant is

$$k^2 = p^2/\hbar^2 = 2m[E - V(\mathbf{r})]/\hbar^2; \tag{43}$$

and the differential equation for the space part from Eqs. (42) then becomes

$$\hbar^2\nabla^2\psi + 2m(E - V)\psi = 0, \tag{44}$$

which is seen to be Schrödinger's famous time-independent equation.

The alternative quantum theory is carried no further here. Those interested are referred to the literature cited.

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Résumé

L'on montre que le principe d'incertitude est simplement une condition pour valider l'approximation de l'optique géométrique; ainsi, il ne peut s'appliquer lorsque des effets d'ondes sont impliqués. En utilisant des évidences scientifiques ordinaires, l'on montre que le principe d'incertitude donne des erreurs de plusieurs ordres de grandeur dans le cas de six exemples explicites empiriques: le microscope optique, l'atome hydrogène, le radio portatif, le microscope à balayage, la désintégration bêta, et les franges de Wiener. La théorie alternative quantique causale qui donne des effets d'onde précis sans incertitude est discutée.

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