Foundations of Physics Letters, Vol. 7, No. 1, 1994

PINCH EFFECT AND AMPERE TENSION TO DRIVE HERING'S PUMP

J. P. Wesley

Weiherdammstrasse 24
W-7712 Blumberg, Germany

Received February 5, 1993; revised June 15, 1993

Contrary to Northrup's claim, the pinch effect, predicted by either the Biot-Savart law or the original Ampere law, accounts for less than half the necessary force to drive Hering's pump. The longitudinal Ampere tension, which is not predicted by the Biot-Savart law, is sufficient to account for the remaining force necessary.

Key words: Ampere force, pinch effect, Hering's pump, Biot-Savart law

1. BACKGROUND

Hering's {1} pump supplies further evidence for the original Ampere {2} force law. A large amount of experimental evidence {1-10} has accumulated over the last 170 years establishing the correctness of Ampere's original force law. The success of Weber {11} electrodynamics, generalized from Ampere's original force law, particularly in explaining the unipolar induction experiments of Müller {12} and Kennard {13}, provides further evidence for the correctness of Ampere's original force law. The success of the Weber-Wesley {14} field theory in predicting the zero self-torque on the Vaughan-Pappas {15} Z-shaped radiating antenna also provides evidence for the original Ampere law.

Despite this evidence, papers {1,16-22} still continue to appear claiming that the original Ampere force law is wrong and that the effects observed can be explained using

0894-9875/94/0200-0095$07.00/0

the Biot-Savart-Maxwell theory. The Biot-Savart law, violating Newton's third law, can be shown to be "absurd" {23}, as it can predict no unique value for the force on Ampere's bridge. Attempts to validate the Biot-Savart law (where a closed current loop does not form a loop that is mechanically closed) involve errors, such as the violation of Newton's third law, that may be readily exposed {14}. For example, Wesley {24} has shown Peoglos' {22} claims to be unjustified. The present analysis of Hering's pump should help to settle the matter in favor of Ampere's original force law.

2. HERING'S PUMP

A diagram of Hering's pump, as constructed for the experiment performed in 1907 by Northrup {1} is presented in Fig. 1. A cylindrical tube of radius R_2 with a stricture of radius R_1 is filled with mercury. When a current I flows (in either direction) the mercury rises to the height h in a small tube provided above. The mercury then forms a fountain or overflows into an appropriate reservoir and is returned via a small hole in the stricture.

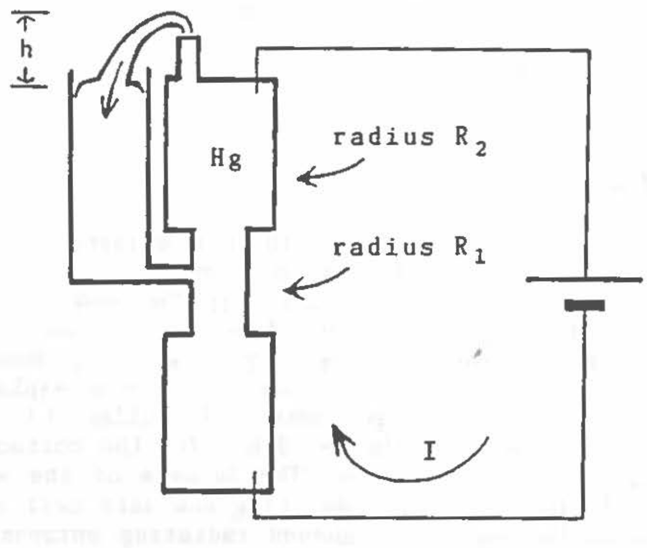

Fig. 1. Diagram of Hering's pump indicating the cylindrical tube of radius R_2 with the stricture of radius R_1. A mercury fountain is produced above when a current I flows.

3. THEORY OF PINCH EFFECT

Neither Hering {3} nor Northrup {1} seemed to have been aware of Ampere's {2} original force law nor Weber {11} electrodynamics; as they do not refer to them nor apply them. Yet the action of Hering's pump can only be explained using Ampere's original force law; and it cannot be explained using the Biot-Savart-Maxwell theory. It is thus, important, even now, 86 years later, to supply the missing theory, that actually explains the action of Hering's pump.

Ampere's {2,14} original force law in terms of volume current densities is given by

$$d^6F_A/d^3rd^3r' = (R/R^3)\{-2J\cdot J' + 3(R\cdot J)(R\cdot J')/R^2\}, \qquad (1)$$

where d^6F_A/d^3rd^3r' is the force on an element of current J in abamp/cm^2 at r due to an element of current J' at r' and $R = r - r'$. This force acting along the line between the current elements and being antisymmetric to an interchange of r and r', obeys Newton's third law. This Ampere force, Eq.(1), may be contrasted with the force prescribed by the Biot-Savart-Maxwell theory:

$$d^6F_B/d^3rd^3r' = J\times(J'\times R)/R^3 = \{-(J\cdot J')R + (R\cdot J)J'\}/R^3, \qquad (2)$$

which clearly violates Newton's third law; as it is not directed along R. The force is always suppose to act perpendicular to the current element J.

Northrup {1} attempted to derive the force driving the mercury in Hering's pump from a "pinch effect" prescribed by the Biot-Savart-Maxwell theory. Forces lateral to the current flow are suppose to give rise to a pressure within the mercury which drives the liquid mercury from a high to a low pressure zone in the axial direction of the device. Northrup uses the effect of the magnetic field produced by an infinitely long cylinder carrying current on the current itself to obtain the following pressure P_B as a function of the radial distance ρ from the axis

$$P_B = (I^2/\pi R_o^4)(R_o^2 - \rho^2), \qquad (3)$$

where I is the total current in abamperes and R_o is the outside radius of the cylinder

This result (3) may also be obtained from direct action at a distance by performing the appropriate integrations of the first term on the right of Eq.(2). The direct integration is to be preferred to the indirect method involving a magnetic field; as it avoids all questions concerning the interpretation of the magnetic field and its effects (For example, can a static magnetic field do work on stationary material?). The radial component of interest giving rise to the pinch effect from the first term on the right of Eq.(2), choosing the point of observation at $z = 0$ and $\phi = 0$, is given by

$$d^6F_{B\rho}/d^3rd^3r' = -\ J^2R_\rho/R^3, \tag{4}$$

where the current density is assumed to be uniform, $J' = J$, and where

$$\begin{aligned} d^3r' &= \rho' d\rho' d\phi' dz', \qquad d^3r = \rho d\rho d\phi dz, \\ R_\rho &= \rho - \rho' \cos\phi', \qquad R^2 = \rho^2 + \rho'^2 - 2\rho\rho' \cos\phi' + z'^2. \end{aligned} \tag{5}$$

Dividing by the lateral area $2\pi\rho dz$, the pressure of interest is given by

$$P_B = (J^2/2\pi)\int_\rho^{R_o} d\rho \int_0^{2\pi} d\phi \int_0^\rho \rho' d\rho' \int_0^{2\pi} d\phi' \int_{-\infty}^{\infty} dz' R_\rho/R^3. \tag{6}$$

Substituting Eqs.(5) into (6) and performing the indicated integrations again yields the result (3).

If the pressure P_B also acts in the axial direction, the net average axial pressure over the transverse area of the cylinder is given from Eq.(3) by

$$\overline{P}_B = (1/\pi R_o^2)\iint P_B d\phi\, \rho d\rho \ = \ I^2/2\pi R_o^2. \tag{7}$$

The difference in pressure between that in the cylinder of radius R_2 and that in the stricture of radius R_1, that can presumably drive the mercury in the pump, is then given by

$$\Delta P_B = \xi g h = (I^2/2\pi)(1/R_1^2 - 1/R_2^2), \tag{8}$$

where ξ is the density of the mercury, g is the acceleration of gravity, and h is the maximum height to which the mercury can rise above the level in the reservoir. Since the Biot-Savart force acts perpendicular to the current density J; this pinch effect given by Eq.(8) is the only effect that can drive the mercury in Hering's pump according to the Biot-Savart law.

Ampere's original force law (1) yields precisely the same pinch pressure, Eq.(3) and the same average pressure, Eq.(7), as the Biot-Savart law (2), which may be readily verified by performing the same integrations indicated in Eq.(6), using both terms in the bracket of Eq.(1). This agreement between the two laws for the transverse force acting is, of course, to be expected.

4. THEORY FOR AMPERE LONGITUDINAL TENSION

In contrast to the Biot-Savart law (2), which predicts no longitudinal force in the direction of the current, Ampere's original force law (1) predicts such a longitudinal force, which helps to account for the force necessary to drive Hering's pump.

To make it clear that the longitudinal Ampere tension arises primarily from two current carrying conuctors in contact the longitudinal force acting on a cylinder of length M/2 and radius R_o, carrying a current density J in contact with a coaxial cylinder of the same length and radius carrying the same current may be considered. For this case Eq.(1) yields the force in the z direction

$$F(\text{cylinder}) = J^2\int_o^R \rho d\rho \int_o^{2\pi} d\phi \int_o^{M/2} dz \int_o^R \rho' d\rho' \int_o^{2\pi} d\phi' \int_{-M/2}^{o} dz' \times (z - z')(1/R^3 - 3Q^2/R^5) , \quad (9)$$

where R^2 is given by the last of Eqs.(5) and

$$Q^2 = \rho^2 + \rho'^2 - 2\rho\rho' \cos(\phi - \phi'). \quad (10)$$

Assuming that $Q^2/M^2 < \pi R_o^2/M^2$ is small, then Eq.(9) yields

$$F(\text{cylinder}) = I^2(-1 - ln2 + lnM) - J^2 \int\int\int\int ln Q. \quad (11)$$

Since the ϕ' integration can be carried out over any 2π interval; the integration over ϕ' yields from the tables of integrals [25]

$$\int_o^{2\pi} d\phi' \, ln(\rho^2 + \rho'^2 - 2\rho\rho'\cos\phi') = \quad (12)$$

$$\pi ln\left[\frac{\rho^2 + \rho'^2 - \sqrt{(\rho^2+\rho'^2)^2 - 4(\rho\rho')^2}}{2}\right] = \begin{cases} 2\pi\, ln\,\rho & \text{for } \rho > \rho', \\ 2\pi\, ln\,\rho & \text{for } \rho < \rho'. \end{cases}$$

Substituting Eq.(12) into (11) and completing the remaining integrations yields

$$F(\text{cylinder}) = I^2 \left[-3/4 - \ln 2 + \ln (M/R_o)\right]. \qquad (13)$$

This result (13) (for R_o small) is seen to be primarily a result of the interaction of the two current carrying cylinders in the immediate neighborhood of the contact, as given by the integral over $\ln Q$ in Eqs.(11) and (12). Comparing this tension with the average pinch force given by πR_o^2 times Eq.(7), it is clear that the dominant force acting can be the Ampere tension rather than the pinch effect.

If M were allowed to go to infinity in Eq.(13) (as assumed above in calculating the pinch force, Eq.(6)) the longitudinal Ampere force would go to infinity. This cannot occur in fact; because the electrical circuit has to be closed, which means that M must remain finite and other forces due to to the remaining portion of the circuit also act on the cylinder.

To obtain a rough estimate of the longitudinal Ampere tension or stress in the mercury in Hering's pump due to the entire circuit the force on Ampere's bridge may be considered, where the circuit is a circle of radius W and the bridge forms half of this circle. From the symmetry in this case the electromagnetic body forces acting on any small portion of the circle must be the same anywhere in the circuit. If the electromagnetic body forces create a tension τ per unit length of the circle, then the net force on the half-circle bridge is given by

$$F(\text{bridge}) = 2W\int_o^{\pi/2} \tau \sin\phi \, d\phi. \qquad (14)$$

Since τ is a constant independent of ϕ; the tension per unit length created by the electromagneitc body forces is given in terms of the force on Ampere's bridge by

$$\tau = F(\text{bridge})/2W. \qquad (15)$$

(It is a matter of indifference here if this tension τ is assumed to be a result of longitudinal or lateral forces.)

In the mercury in Hering's pump the only forces that can act are the electromagnetic and gravitational body forces on the mercury itself. The rigid boundaries can play only a passive role. For a cylinder of length M forming a small portion of the total (circular) circuit the total tension developed by the electromegnetic body forces may be approximated by

$$M = F(\text{bridge})M/2W. \tag{16}$$

Since the force on a semi circular Ampere bridge in a circular circuit is difficult to derive mathematically and it has never been measured; it is sufficient here for a rough estimate to consider a rectangular circuit of sides P and L, where the movable bridge consists of the side L with legs as portions of the two sides of length P (the force being independent of the length of the legs) [26]. Using Eq.(13) for the case M = P and carrying out the remaining integrations, the force on Ampere's bridge for conductors of circular cross section is then given by

$$F(\text{bridge}) = 2I^2\Big[-3/4 + \ln 2 + \ln(L/R_o) + \sqrt{1 + L^2/P^2} - \ln\big(1 + \sqrt{1 + L^2/P^2}\big)\Big]. \tag{17}$$

In ignorance of the precise geometry of Northrup's entire circuit a rough estimate is given by considering a square circuit, where P = L. In this case Eq.(17) becomes

$$F(\text{bridge}) = 2I^2\big[C + \ln(L/R_o)\big], \tag{18}$$

where

$$C = \sqrt{2} + \ln 2 - \ln(\sqrt{2} + 1) - 3/4 = 0.4760\ldots\,. \tag{19}$$

Replacing the radius W of the circular circuit by the side of a square circuit L such that the areas of the circle and square are the same, $W = L/\sqrt{\pi}$, Eq.(16) yields an estimate of the longitudinal Ampere stress in a mercury cylinder of length M; thus,

$$P(\text{longitudinal}) = M/\pi R^2 = (MI^2/\sqrt{\pi}R_o^2L)\big[C + \ln(L/R_o)\big], \tag{20}$$

where C is given by Eq.(19) where R_o is the radius of the cylinder.

Including the pinch effect, Eq.(8), the net pressure available to drive Hering's pump according to Ampere's original force law (1) is roughly estimated to be

$$\Delta P_A = (I^2/2\pi R_1^2)\left\{1 + 2\sqrt{\pi}(M_1/L)\left[0.4760 + \ln(L/R_1)\right]\right\} \quad (21)$$
$$- (I^2/2\pi R_2^2)\left\{1 + 2\sqrt{\pi}(M_2/L)\left[0.4760 + \ln(L/R_2)\right]\right\},$$

which may be compared with Eq.(8) for the pinch effect alone. This result (21) may also be compared with a prior estimate [27].

The energy to raise the mercury continuously in Hering's pump is apparently derived from the same energy source as that required to sustain the current I against ohmic losses.

5. EXPERIMENTAL RESULTS AND CONCLUSIONS

Northrup [1] reports that for $R_2 = 1.27$ cm, $R_1 = 0.635$ cm, and $I = 180$ abamp that the mercury rose to a maximum height of

$$h(\text{observed}) = 1.524 \text{ cm}. \quad (22)$$

According to formula (8), where the density of mercury is $\xi = 13{,}546$ gm/cm^3 and the acceleration of gravity is $g = 980.0$ cm/sec^2, the maximum height predicted by the Biot-Savart-Maxwell theory should have been

$$h(\text{Biot-Savart-Maxwell}) = 0.723 \text{ cm}. \quad (23)$$

This predicted height is, thus, less than half the observed height. Due to experimental imperfections the observed height should be less than the ideal height predicted by theory. Since no force other than the pinch force, leading to the prediction (23), is available to drive the mercury in Hering's pump according to the Biot-Savart-Maxwell theory; the Biot-Savart-Maxwell theory fails.

From Northrup's [1] Fig. 6 the height of the mercury cylinders are estimated to be $M_1 = 2.54$ cm and $M_2 = 3.36$ cm; and the dimension of the circuit is roughly chosen as $L = 18.3$ cm. From Eq.(21) the height the mercury should rise according to Ampere's law (1) is roughly estimated to be

$$h(\text{Ampere}) = 2.048 \text{ cm}. \quad (24)$$

Although no numerical agreement can be expected here in ignorance of the precise geometry of Northrup's setup and in view of the approximations used; this result (24) does indicate that the Ampere theory, which includes both a pinch effect and a longitudinal tension can account for the force necessary to drive Hering's pump.

REFERENCES

1. E. F. Northrup, *Phys. Rev.* 24, 474 (1907).
2. A. M. Ampere, *Mem. Acad. R. Sci.* 6, 175 (1923); *Memoires sur l'Electrodynamique* (Gauthier Villars, Paris, 1882) Vol. I, p. 25.
3. C. Hering, *J. Franklin Inst.* 171, 73 (1911); 192, 599 (1921); *Trans. Am. Inst. Elect. Eng.* 42, 311 (1923).
4. F. F. Cleveland, *Phil. Mag. Suppl.* 7, 21, 416 (1936).
5. I. A. Robertson, *Phil. Mag.* 36, 32 (1945).
6. P. Graneau, *J. Appl. Phys.* 53, 6648 (1982); 55, 2598 (1984); 62, 3006 (1987); *Nature*, 295, 311 (1982); *Phys. Lett.* 97A, 253 (1983); 107A, 235 (1985); *IEEE Trans. Magn.* MAG-20, 444 (1984); *Appl. Phys. Lett.* 46, 468 (1987).
7. P. T. Pappas and P. G. Moyssides, *Phys. Lett.* 111A, 193 (1985).
8. P. G. Moyssides and P. T. Pappas, *J. Appl. Phys.* 59, 19 (1986).
9. T. E. Phipps, Jr., *Phys. Essays*, 3, 198 (1990).
10. T. E. Phipps and T. E. Phipps, Jr., *Phys. Lett. A*, 146, 6 (1990).
11. W. E. Weber, *Abh. Leibnizens Gez., Leip.* 316 (1846); *Ann. der Phys.* 73, 229 (1848); *Wilhelm Weber's Werke* (Julius Springer, Berlin, 1893).
12. F. J. Müller, in *Progress in Space-Time Physics 1987*, ed. J. P. Wesley (Benjamin Wesley, W-7712 Blumberg, Germany, 1987) pp. 156-169.
13. E. Kennard, *Phil. Mag.* 33, 179 (1917).
14. J. P. Wesley, *Found. Phys. Lett.* 3, 443, 471, 641 (1990); *Advanced Fundamental Physics* (Benjamin Wesley, W-7712 Blumberg, Germany, 1991) pp. 212-272.
15. P. T. Pappas and T. Vaughan, *Phys. Essays*, 4 (1991); and P. T. Pappas in *Proc. Int. Conf. Foundations of Mathematics and Physics, Perugia, 1989*, eds. U. Bartocci and J. P. Wesley (Benjamin Wesley, W-7712 Blumberg, Germany, 1990) pp. 203-214.
16. J. C. Maxwell, *A Treatise on Electricity and Magnetism* (Clarendon Press, Oxford, 1891) reprint (Dover, New York, 1952) Vol. 2, Art. 686, pp. 318-320.
17. C. Christodoulides, *Am. J. Phys.* 56, 357 (1988); *J. Phys. A: Math. Gen. Phys.* 20, 2037 (1987).

18. J. G. Ternan, *J. Appl. Phys.* 57, 1743 (1985); *Phys. Lett. A*, 115,
19. A. M. Hillas, *Nature*, 302, 271 (1983).
20. K. H. Carpenter, *IEEE Trans. Magn.* MAG-20, 2159 (1984).
21. A. E. Robson and J. D. Sethian, *Am. J. Phys.* 60, 111 (1992).
22. V. Peoglos, *J. Phys. D: Appl. Phys.* 21, 1055 (1988).
23. J. P. Wesley, *Bull. Am. Phys. Soc.* 28, 1310 (1983).
24. J. P. Wesley, *J. Phys. D: Appl. Phys.* 22, 849 (1989).
25. C. D. Hodgman, ed., *Handbook of Chemistry and Physics* (Chemical Rubber, Cleveland, 1961) p. 284, Eq.(453).
26. J. P. Wesley, *Found. Phys. Lett.* 3, 433 (1990). Eq.(27) gives the force on Ampere's bridge for wires of rectangular cross section.
27. J. P. Wesley, *Found. Phys. Lett.* 3, 433 (1990), Eq.(40). The pressure to drive Hering's pump and the oscillations in Phipps' wedge, as given by Eq.(40), assumes conductors of square (instead of circular) cross section. This result is also in error; because the rigid portions of the circuit are assumed to play an active role; whereas only the body forces in the mercury itself can contribute to the pressures observed. Moreover, the pinch effect was overlooked and is not included in this result (40).