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WEBER ELECTRODYNAMICS, PART II UNIPOLAR INDUCTION, Z-ANTENNA

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Weber electrodynamics predicts the localized unipolar induction observed by Müller and Kennard; whereas the Maxwell theory, based upon closed current loops and the flux rule, fails. The Weber theory for high frequency fields predicts a zero self torque on the Pappas-Vaughan Z-antenna, as observed. In contrast, the Maxwell theory predicts a sizeable self torque which is not observed.

Key words: electrodynamics, Weber versus Maxwell, unipolar induction, Z-antenna self torque.

1. INTRODUCTION

This is part II of a paper presented in three parts. Part I (*Found. Phys. Lett.* in this issue) presents the Weber theory extended to fields and radiation. It presents the steady-current experimental evidence for the original Ampere law and Weber electrodynamics and the failure of the Biot-Savart law and Maxwell theory. The present part II shows that the Weber theory predicts the localized unipolar induction experiments of Müller and Kennard; while the Maxwell theory again fails. High frequency field effects are also correctly predicted by the Weber electrodynamics: The observed zero self torque on the Pappas-Vaughan Z-antenna is predicted by Weber electrodynamics; while the Maxwell theory predicts a sizeable self torque which is not zero and which is not observed. Part III concerns the impact

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of Weber electrodynamics on mechanics and gravitation.

2. WEBER THEORY OF UNIPOLAR INDUCTION AND THE EXPERIMENTS OF MÜLLER AND KENNARD

Much confusion exists today concerning unipolar induction, because the traditional theories of Faraday and Maxwell give equivocal answers or no answers at all. These theories do not agree with the important experimental result of Müller [1] and Kennard [2]. The Weber theory, on the other hand, being based upon the force between point charges, yields unequivocal agreement with all of the experimental results.

The only induction law provided by Maxwell theory is Faraday's law of electromagnetic induction,

$$\text{emf} = - \partial\Phi/\partial t c, \tag{1}$$

where emf is the electromotive force induced in a closed loop and Φ is the magnetic flux through the loop. It is frequently attempted to use this Maxwell's flux rule (1) for all induction phenomena; but this type of induction is limited to the case of a *changing* magnetic flux through a *closed* loop produced by *closed* current loop sources. The more general Weber theory predicts induction where no magnetic flux can be defined and no *closed* current loops at all need be involved. For example, unipolar induction involves no change in magnetic flux (which remains zero) through the loop in which current is induced, as recognized by Cohn [3], Culliwick [4], and Feynman [5] (Although others, such as Savage [6], Panofsky and Phillips [7], and Scanlon et al [8] try to see a change in flux.). The Maxwell flux rule, involving the net flux through a closed loop, either assumes that the induced emf occurs uniformly around the loop; or else it fails to predict *where* the seat of the emf might be in the closed loop. Müller reveals the fact that the seat of the emf can be localized experimentally and that its position in the closed loop can be determined.

Faraday [9] performed his famous rotating disk experiment in 1832. A copper disk is rotated near the pole of a magnet. Stationary wires touch the center and the rim of the disk through sliding contacts, as shown in Fig. 1. This produces an emf, which can be detected by inserting a volt meter in the circuit.

Faraday attributed this "motional emf" to the disk

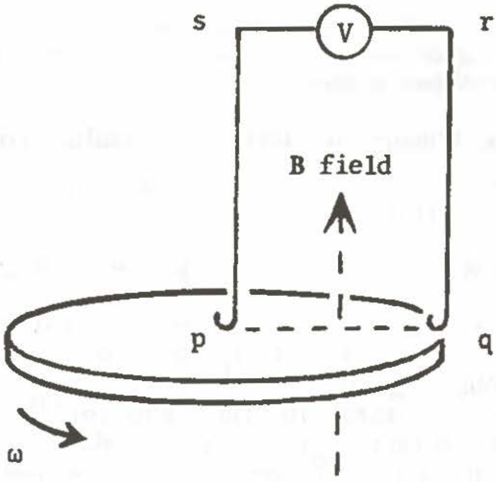

Fig. 1. Faraday's rotating disk experiment. The magnetic field **B** is perpendicular to the disk. The induced emf is registered on the volt meter.

"cutting" magnetic field lines, the induced electric field at a point on the disk being given by

$$\mathbf{E} = \mathbf{v} \times \mathbf{B}/c, \qquad (2)$$

where **v** is the velocity of the disk and **B** is the magnetic field at the point in question. Faraday originally assumed that the magnetic field lines were rigidly fixed to the magnet; and, thus, relative motion between the disk and the source of the magnetic field was needed to generate an emf. This view is still found in most textbooks and held by many physicists, such as Trocheris [10] and Cullwick [4]. But it is not true. When the magnet is rotated with the disk, precisely the same emf is induced; as soon discovered by Faraday himself. Faraday then changed his mind: He decided that the magnetic field lines remain fixed in space; even though the magnet itself rotated. In this way the "cutting" hypothesis could still work. In 1851 Faraday [11] again changed his mind: He decided that the magnetic field lines did, in fact, rotate with the magnet after all. The "moving" magnetic field lines, "cutting" the stationary external circuit rs in Fig. 1, generated the observed emf. Cullwick [4] agrees with this view of Faraday. He says that the emf occurs in that wire which is in motion relative to the magnet. This conclusion is not supported by Müller's experiment nor Weber's theory.

2.1. Weber theory of unipolar induction

The Weber force on a moving charge q at $\mathbf{r}$ due to a moving charge q' at $\mathbf{r}'$ is

$$c^2\mathbf{F}_W = (qq'\mathbf{R}/R^3)\left[c^2 + V^2 - 3(\mathbf{V}\cdot\mathbf{R})^2/2R^2 + \mathbf{R}\cdot d\mathbf{V}/dt\right], \quad (3)$$

where $\mathbf{R} = \mathbf{r} - \mathbf{r}'$ and $\mathbf{V} = \mathbf{v} - \mathbf{v}'$ are the relative position and relative velocity. Induction involves the force on the mobile electrons $- q_e$ in a detector conductor. For unipolar induction the electrons in the detector have only the velocity of the detector itself $\mathbf{v}_i$ (Conduction currents in the detector are not considered.). For unipolar induction only steady current sources are involved where $d\mathbf{v}_e'/dt = 0$. The net force on the detector electrons is then the sum of the force due to the source ions q_i' moving with a velocity $\mathbf{v}_i'$, which is simply the velocity of the source conductor, plus the force due to the source electrons $- q_e'$ moving with a velocity $\mathbf{v}_e' + \mathbf{v}_i'$, where $\mathbf{v}_e'$ is the steady electron velocity relative to the source conductor. For the case of the source carrying no net charge, $q_i' = q_e'$, Eq.(3) yields the Weber force on the detector electrons as

$$\begin{aligned} c^2\mathbf{F}_W = (q_e q_e'\mathbf{R}/R^3)\big[&-2\mathbf{v}_i\cdot\mathbf{v}_e' + 3(\mathbf{R}\cdot\mathbf{v}_i)(\mathbf{R}\cdot\mathbf{v}_e')/R^2 + v_e'^2 \\ &+ 2\mathbf{v}_e'\cdot\mathbf{v}_i' - 3(\mathbf{R}\cdot\mathbf{v}_e')(\mathbf{R}\cdot\mathbf{v}_i')/R^2 - 3(\mathbf{R}\cdot\mathbf{v}_e')^2/2R^2\big]. \end{aligned} \quad (4)$$

The last four terms on the right of Eq.(4) involve the force due to velocity squared currents on a static charge. Such effects are very small and can be ordinarily ignored (These terms are considered in part III.) The observable unipolar induction between point charges then becomes

$$c^2\mathbf{F}_W = (q_e q_e'\mathbf{R}/R^3)\left[-2\mathbf{v}_i\cdot\mathbf{v}_e' + 3(\mathbf{R}\cdot\mathbf{v}_i)(\mathbf{R}\cdot\mathbf{v}_e')/R^2\right]. \quad (5)$$

An interesting feature of this result (5) is that the motion of the source conductor $\mathbf{v}_i'$ does not enter in! Only the velocity of the source electrons relative to the source conductor $\mathbf{v}_e'$ is involved. It may be noted that moving a current carrying wire parallel to itself with the velocity $\mathbf{v}_i'$ does not change the net current in the wire, the electron current $- q_e'(\mathbf{v}_e' + \mathbf{v}_i')$ plus the ion current $q_i'\mathbf{v}_i'$ yielding $- q_e'\mathbf{v}_e'$ when $q_e' = q_i'$.

When *extended* sources are moved with a velocity $\mathbf{v}_i'$ an additional effect, a "pseudo-effect", occurs, which yields the appearance of a time rate of change of current or the acceleration of charges due to a variation in the electromagnetic field at the detector with time. This force per unit charge is given by

$$- (\mathbf{v}_i' \cdot \nabla)(\mathbf{A} - \nabla \Gamma). \tag{6}$$

It vanishes for point charges. For extended sources this result (6) must be added to Eq.(5) for the unipolar induced force per unit charge given in terms of electromagnetic fields; thus, using Eqs.(I.11) and (I.12) (where I refers to equations in part I of this three part paper)

$$\begin{aligned} c\mathrm{E}(\text{induction}) = \mathbf{v}_i \times (\nabla \times \mathbf{A}) - \mathbf{v}_i \nabla \cdot \mathbf{A} \\ + (\mathbf{v}_i \cdot \nabla)\nabla \Gamma - (\mathbf{v}_i' \cdot \nabla)(\mathbf{A} - \nabla \Gamma). \end{aligned} \tag{7}$$

This Weber-Wesley result (7) can predict the induced electric field in a detector for many possible situations; but the experiments of interest here require only closed loop current sources where $\nabla \cdot \mathbf{A} = \Gamma = 0$. In addition, in these experiments the motion of the source $\mathbf{v}'$ is confined to the situations where $- (\mathbf{v}' \cdot \nabla)\mathbf{A} = 0$. For the experimental situations of interest here the unipolar induction reduces to

$$\mathbf{E} = \mathbf{v} \times (\nabla \times \mathbf{A})/c. \tag{8}$$

Using the fact that $\mathbf{B} = \nabla \times \mathbf{A}$, Eq.(8) yields the original Faraday result (2). It might, thus, seem that the Weber theory offers nothing more than the Faraday theory; but this is not true. The derivation of Eq.(2) from the Weber theory presented here now makes the meaning of the magnetic field clear. The interpretation of the magnetic field by Faraday and Maxwell as physically tangible rigid lines of force attached to a source is seen to be physically untenable. The **B** field, like the **A** field from which it is defined, is merely a mathematical artifact, a mathematical device, of no particular direct physical significance, used to help solve the problem of how moving point *source* charges affect moving *detector* charges.

To make it abundantly clear that magnetic field lines can never "move" to "cut" a stationary wire, it may be noted from Eqs.(5) and (8) that the induced electric field is *entirely independent* of the state of rotational motion of source solenoid (or permanent magnet) about the axis of the solenoid. Rotating the solenoid merely moves the wires of the solenoid parallel to themselves; so, as explained following Eq.(5) above, the net current in the solenoid remains the same.

The question remains: What is the frame of reference in which the velocity $\mathbf{v}_i$ is to be measured? It is not to be measured with respect to the current electrons in the source solenoid nor with respect to the moving solenoid itself. Experimentally the velocity $\mathbf{v}_i$ is measured with respect to the laboratory. When the disk is stationary in the laboratory no induced electric field is observed. It may be seen that Eq.(5), which yields the induction formula (8), arises from the cross product terms of the squares of *relative* velocities: $(\mathbf{v}_i - \mathbf{v}'_e - \mathbf{v}'_i)^2$, $|\mathbf{R}\cdot(\mathbf{v}_i - \mathbf{v}'_e - \mathbf{v}'_i)|^2$, $(\mathbf{v}_i - \mathbf{v}'_e)^2$ and $|\mathbf{R}\cdot(\mathbf{v}_i - \mathbf{v}'_i)|^2$. The squared terms drop out leaving only the cross product terms $2\mathbf{v}_i\cdot\mathbf{v}'_e$ and $2(\mathbf{R}\cdot\mathbf{v}_i)(\mathbf{R}\cdot\mathbf{v}'_e)$. Thus, the Weber theory starts out using only *relative* velocities to derive a result which has only an *absolute* velocity (The reference frame is experimentally the laboratory.). The same thing happens in deriving Ampere's law from the Weber theory (Eq.(I.5) from (I.3) and (I.4)). The cross product terms yield a result in which the source and detector electron velocities become independently prescribed (experimentally with respect to the laboratory).

1.2. Unipolar induction experiments of Kennard and Müller

Kennard [2] eliminated the circuit pqrs, as shown in Fig. 1. No current flow was involved. He measured directly the static voltage difference induced across pq. There was no doubt that the seat of the emf was across pq. Since the static voltage difference is extremely small; the effect was enhanced by introducing a capacitor across pq. The capacitor consisted of two concentric cylinders. They were connected by a radial wire which functioned as the radius pq of the Faraday disk, Fig. 1. The magnetic field was not produced by a permanent magnet but by a concentric solenoid outside the capacitor. The solenoid was free to rotate independently. With this setup Kennard observed the following: 1) A voltage difference was induced when the radial wire together with the capacitor were rotated and the current carrying solenoid was stationary. 2) No voltage was induced when the solenoid was rotated and the radial wire with the capacitor were stationary. And 3) the same voltage difference as in case 1) above was generated when the radial wire with the capacitor and the solenoid were all rotated together at the same rate as in case 1). Kennard, thus, demonstrated that unipolar induction occurred when there was no relative motion whatso-ever between any portions of the apparatus. The induced voltage depended in this case only upon the *absolute* rotational velocity with respect to the laboratory of the whole apparatus as a unit. More precisely, in agreement with the Weber theory above, the induced voltage was only a function of the rotational velocity with respect to the laboratory of the radial wire with the capacitor and was independent of the rate of rotation of the solenoid. Kennard's result clearly shows that magnetic field lines do not rotate with the solenoid as assumed by Faraday in 1851.

Müller |1| obtained the same result as Kennard using a permanent magnet and the setup shown in Fig. 2. In addition, Müller was able to localize the seat of the emf. Like Kennard, Müller replaced Faraday's disk with a straight wire pq. Instead of having the equipment perform complete rotations, Müller simply oscillated the various portions of his setup back and forth. The portion pq, the portion rs, and the magnet could be oscillated independently.

Considering Fig. 2, if portion pq is oscillated rotationally back and forth rapidly in comparison to the RC decay time of the circuit, while the portion rs is held stationary, an emf 1 will be induced across pq and none will be induced across rs. This will cause an oscillating voltage V_1 to appear across R_1 and essentially no voltage signal across R_2. When rs is rotated while pq remains stationary a signal V_2 will appear across R_2 and essentially no signal across R_1, indicating an emf is induced in rs and none in pq. In this way he was able to distinguish in which branch of the circuit qpt or qrst an emf arose. The seat of the emf in the closed loop pqrstp could, thus, be localized.

To eliminate the possibility that when the magnet is oscillated "moving" magnetic field lines might also induce an emf in the capacitor branch of the circuit giving spurious results the experiment was also performed using an iron yoke around the magnet extending outward and inward to the plane of Fig. 2. With the yoke most of the magnetic field remains in the yoke; and the wire qrs and capacitor branch of the circuit were sheilded from the magnetic field. With the yoke no magnetic field existed in the capacitor branch; and no emf could possibly be induced in this branch.

The experimental results are summarized in Table 1. The symbol - signifies no angular oscillation and the symbol ω signifies an angular oscillation, where, if two or more portions were oscillated at the same time, they were

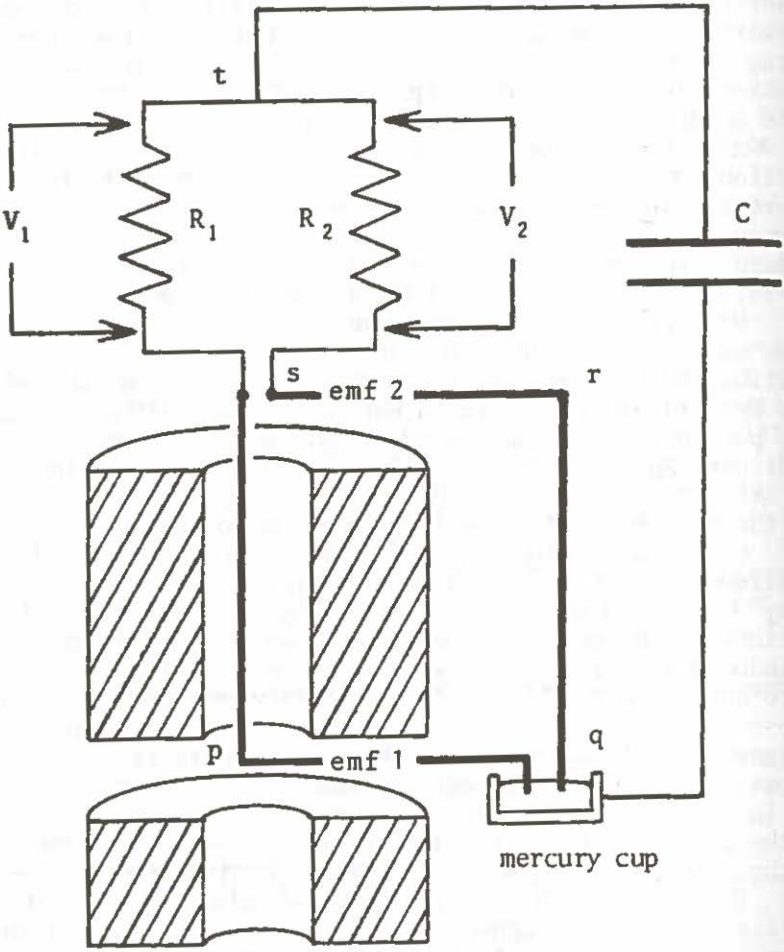

Fig. 2. Diagram of the Müller experiment to determine the seat of unipolar induction using an annular shaped permanent magnet (shaded) with a gap as shown. Portions pq and rs of the circuit and the magnet can be oscillated back and forth independently by virtue of the mercury cup contact. An oscillating voltage V_1 across R_1 indicates an emf 1 induced in the portion pq; and an oscillating voltage V_2 across R_2 indicates an emf 2 induced in the portion rs.

Table 1. Seat of unipolar induced emf for different cases

case	oscillating			no yoke		with yoke	
	magnet	wire pq	wire rs	emf pq	emf rs	emf pq	emf rs
1	-	-	-	0	0	0	0
2	-	-	ω	0	+	0	0
3	-	ω	-	+	0	+	0
4	ω	-	-	0	0	0	0
5	-	ω	ω	cancelled		+	0
6	ω	-	ω	0	+	0	0
7	ω	ω	-	+	0	+	0
8	ω	ω	ω	cancelled		+	0

mechanically coupled together to oscillate as a unit. The symbol + means an emf was observed and the symbol 0 means that no emf was observed. For the cases 5 and 8 no signal was observed; as the emf's in pq and rs, being the same, acted like two batteries back to back, which prohibited any current from flowing and any voltage from being registered.

1.3. Discussion concerning unipolar induction

The experimental results of Kennard and those of Müller, as summarized in Table 1, agree in all particulars with the Weber theory. Their results do not agree in all particulars with the theories of Maxwell and Faraday.

The Maxwell flux rule does not work; as the amount of flux through the loop pqrstp remains zero for all cases; and the emf is localized and not uniformly distributed around the whole loop.

The fact that unipolar induction depends solely upon the *absolute* (or more precisely the *laboratory*) rotational velocity of the detector and does not depend at all upon the rotational velocity of the source of the magnetic field contradicts the usual traditional Faraday theory that induction arises only by virtue of the *relative* rotational motion of the source and the detector.

2. DETERMINATION OF SELF TORQUE ON PAPPAS-VAUGHAN Z-ANTENNA

The Maxwell field theory displaced the Weber action at a distance theory toward the end of the last century; because the Maxwell theory predited Herz electromagnetic waves, and the Weber theory could not. The failures of the Maxwell theory for slowly varying effects, in contrast to the success of the Weber theory, (as discussed above) was simply "forgotten". But the situation has now changed dramatically. The Weber theory is no longer merely an action at a distance theory; it is now also a *field theory*; and it now also predicts *electromagnetic waves*. The question then arises: Does the superiority of the Weber theory for slowly varying effects also hold true for rapidly varying effects involving time-retarded fields? Certainly the Weber field theory is more complicated with its two additional field potentials Γ and $\mathbf{G}$, Eqs.(I.12); so one might expect it to provide some advantages. The present problem of predicting the self-torque on the Pappas-Vaughan Z-antenna provides an excellent test case. The Weber field theory predicts the correct observed zero self torque; whereas the Maxwell field theory predicts a sizeable nonvanishing self torque, which is not observed. The superiority in general of the Weber theory for rapidly varying time retarded fields compared with the Maxwell theory is thereby proven. It can, of course, be expected that the Weber theory will give precisely the same answers as the Maxwell theory for appropriate limited situations.

For slowly varying effects the biggest failure of the Maxwell theory is its violation of Newton's third law, being based upon the Biot-Savart law or the Lorentz force law. Does the same failure occur for rapidly varying field effects? The present Z-antenna example provides the answer: The Maxwell theory continues to violate Newton's third law for rapidly varying time retarded field effects. This violation of Newton's third law by the Maxwell theory is independent of the fact that Newton's third law for time retarded fields is not in general obeyed *instantaneously* unless momentum is assigned to the field itself. Time retarded fields (such as needed to represent light waves) take on physical properties of their own, transmitting energy and momentum, independent of the original source or final sink. However, for the present Z-antenna example these inertial properties of the field are automatically taken into account; and the Maxwell theory still violates Newton's third law for the time average self torque.

The Weber and Maxwell theories considered here involve the potential fields (Eqs.(I.14) and (I.15)) defined by

$$\Phi = \int d^3r'\rho'(\mathbf{r}',t^*)/R, \qquad \mathbf{A} = \int d^3r'\mathbf{J}'(\mathbf{r}',t^*)/cR,$$
$$\Gamma = \int d^3r'\mathbf{R}\cdot\mathbf{J}'(\mathbf{r}',t^*)/cR, \qquad \mathbf{G} = \int d^3r\, \mathbf{R}\rho'(\mathbf{r}',t^*)/R, \tag{9}$$

where t* is the retarded time,

$$t^* = t - R/c. \tag{10}$$

The Weber force (Eq.(I.11)) is given by

$$d^3\mathbf{F}_W/d^3r = -\rho\nabla\Phi + \mathbf{J}\times(\nabla\times\mathbf{A})/c - \rho\partial\mathbf{A}/\partial tc - \mathbf{J}\,\nabla\cdot\mathbf{A}/c$$
$$+ (\partial\mathbf{J}/\partial t)\Phi/c^2 + (\mathbf{J}\cdot\nabla)\nabla\Gamma/c + \rho\nabla\partial\Gamma/\partial tc - \left[(\partial\mathbf{J}/\partial t)\cdot\nabla\right]\mathbf{G}/c^2. \tag{11}$$

The Maxwell-Lorentz force (Eq.(I.19)) is given by

$$d^3\mathbf{F}_M/d^3r = -\rho\nabla\Phi - \rho\partial\mathbf{A}/\partial tc + \mathbf{J}\times(\nabla\times\mathbf{A})/c. \tag{12}$$

These Eqs.(9) through (12) are used below to determine the theoretically expected self torque on the Pappas-Vaughan Z-antenna, as predicted by the Weber theory, Eqs.(9), (10), and (11) and as predicted by the Maxwell theory, the first two of Eqs.(9), (10) and (12).

2.1. Pappas-Vaughan experiment with Z-antenna

Pappas and Vaughan [12] suspended a Z-shaped antenna, as shown in Fig. 3 by a 5 m long nylon fiber. No other mechanical connection to the antenna is involved. The antenna is driven inductively by an air core transformer at the center at a frequency ($\approx$ 150 Mhz) such that the standing electromagnetic waves along the antenna have a wavelength λ ($\approx$ 2 m) matching the dimensions of the antenna as shown in Fig. 3. When driven the antenna shows zero self torque. The torsion balance formed by the suspended antenna is sufficiently sensitive to detect torques of only 10^{-7} Nt m.

The geometry and choice of coordinates are indicated in Fig. 3. The antenna is suspended along the z axis. Portion 1 is taken at $x = \lambda/2$ from $y = 0$ to $y = \lambda/4$, the central portion 2 is taken along the x axis from $x = \lambda/2$ to $x = -\lambda/2$, and the portion 3 is taken at $x = -\lambda/2$ from $y = 0$ to $y = -\lambda/4$.

The air core transformer at the center induces current and charge densities that are time harmonic with all portions

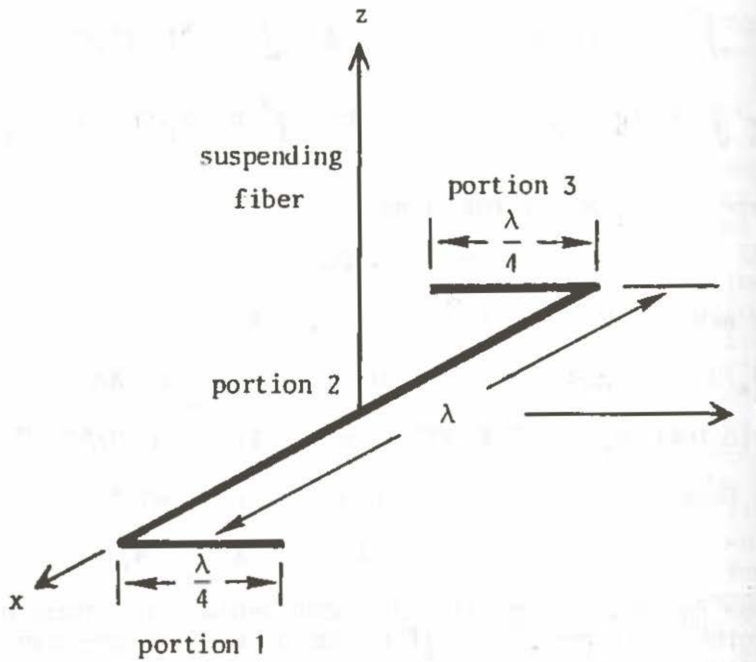

Fig. 3. Diagram of the Pappas-Vaughan Z-shaped antenna suspended by a fiber along the z axis, showing the choice of coordinates and the three portions upon which a standing electromagnetic wave is established. Portions 1 and 3 are a quarter wavelength long, and portion 2 is a wavelength long.

of the antenna in phase; thus,

$$\mathbf{J}(\mathbf{r},t) = \mathbf{J}(\mathbf{r})\cos\omega t, \qquad \rho(\mathbf{r},t) = \rho(\mathbf{r})\sin\omega t, \tag{13}$$

where $\omega = 2\pi f$ is the angular frequency. To satisfy the equation of continuity for charge, $\nabla\cdot\mathbf{J} + \partial\rho/\partial t = 0$, the time harmonic variation for the charge is taken as $\sin\omega t$, when the time variation of the current is taken as $\cos\omega t$. The space part of the current density $\mathbf{J}(\mathbf{r})$ induced in the various portions of the antenna are

$$\begin{aligned} \mathbf{J}_1 &= \mathbf{e}_y\, I\cos ky\,\delta(x - \lambda/2)\,u(y)\,u(-y + \lambda/4), \\ \mathbf{J}_2 &= -\mathbf{e}_x\, I\cos kx\,u(x + \lambda/2)\,u(-x + \lambda/2)\,\delta(y), \\ \mathbf{J}_3 &= \mathbf{e}_y\, I\cos ky\,\delta(x + \lambda/2)\,u(-y)\,u(y + \lambda/4), \end{aligned} \tag{14}$$

where $\delta(x)$ is the delta function, $u(x)$ is the unit step function, zero for $x < 0$ and unity for $x \geq 0$, and I is the peak current. The space part of the charge density $\rho(\mathbf{r})$ induced on the various portions of the antenna are

$$\begin{aligned} c\rho_1 &= I \sin ky\, \delta(x - \lambda/2)\, u(y)\, u(-y + \lambda/4), \\ c\rho_2 &= -I \sin kx\, u(x + \lambda/2)\, u(-x + \lambda/2)\, \delta(y), \\ c\rho_3 &= I \sin ky\, \delta(x + \lambda/2)\, u(-y)\, u(y + \lambda/2). \end{aligned} \tag{15}$$

2.2. Self torque on Z-antenna due to forces obeying Newton's third law are zero

A force $\mathbf{F}(\mathbf{r},\mathbf{r}')$ acting on an unprimed particle (or volume element d^3r) at $\mathbf{r}$ due to a primed particle (or volume element d^3r') at $\mathbf{r}'$ that satisfies Newton's third law is of the form

$$\mathbf{F}(\mathbf{r},\mathbf{r}') = (\mathbf{r} - \mathbf{r}')H(\mathbf{r},\mathbf{r}') = -\mathbf{F}(\mathbf{r}',\mathbf{r}) = -(\mathbf{r}' - \mathbf{r})H(\mathbf{r}',\mathbf{r}), \tag{16}$$

where $H(\mathbf{r},\mathbf{r}') = H(\mathbf{r}',\mathbf{r})$ is a function symmetric to an interchange of primed and unprimed coordinates, and $\mathbf{F}(\mathbf{r}',\mathbf{r})$ is the force acting on the primed particle (or volume element d^3r') at $\mathbf{r}'$ due to the unprimed particle (or volume element d^3r) at $\mathbf{r}$. The torque $T(\mathbf{r},\mathbf{r}')$ about an axis, which may be taken as the z axis, produced by the force $\mathbf{F}(\mathbf{r},\mathbf{r}')$ acting on the unprimed particle (or volume element d^3r) is given, using Eq.(16) by

$$T(\mathbf{r},\mathbf{r}') = \left[\mathbf{s} \times \mathbf{F}(\mathbf{r},\mathbf{r}')\right]\cdot \mathbf{e}_z = \left[x(y - y') - y(x - x')\right]H(\mathbf{r},\mathbf{r}'), \tag{17}$$

where

$$\mathbf{s} = x\mathbf{e}_x + y\mathbf{e}_y, \tag{18}$$

is the radial distance from the z axis, the lever arm, and $\mathbf{e}_x$, $\mathbf{e}_y$, and $\mathbf{e}_z$ are unit vectors in the cartesian coordinate directions. The net self torque on the system, the torque due to the force acting on the unprimed particle plus the torque due to the force acting on the primed particle, then becomes, using Eqs.(16), (17), and (18),

$$\begin{aligned} T(\mathbf{r},\mathbf{r}') + T(\mathbf{r}',\mathbf{r}) &= \left[\mathbf{s} \times \mathbf{F}(\mathbf{r},\mathbf{r}') + \mathbf{s}' \times \mathbf{F}(\mathbf{r}',\mathbf{r})\right]\cdot \mathbf{e}_z \\ &= (yx' - xy')H(\mathbf{r},\mathbf{r}') + (y'x - x'y)H(\mathbf{r}',\mathbf{r}) = 0 \end{aligned} \tag{19}$$

A summation over all particle pairs or an integration over both primed and unprimed volume elements then also yields zero.

This result was, of course, to be expected; since, according to Newton's third law for rotation, the sum of the internal torques acting on *any* isolated system must be zero.

2.3. Weber theory predicts a zero self torque on the Pappas-Vaughan Z-antenna

Substituting Eqs.(13) into Eqs.(9) through (11), the Weber force on an element of volume d^3r at $\mathbf{r}$ due to a volume element d^3r' at $\mathbf{r}'$ may be obtained at any instant. Experimentally only the time average force is of interest here. The average over a cycle involves the integrals

$$\begin{aligned} \int_0^{2\pi/\omega} dt \sin\omega t \sin(\omega t - kR) &= (\cos kR)/2, \\ \int_0^{2\pi/\omega} dt \cos\omega t \cos(\omega t - kR) &= (\cos kR)/2, \end{aligned} \tag{20}$$

where $k = \omega/c = 2\pi/\lambda$ is the propagation constant.

Using this result (20) the time average force between volume elements from Eqs.(9) through (11), which is appropriate for the Pappas-Vaughan Z-antenna, is given by

$$\begin{aligned} 2c^2 \left\langle d^6\mathbf{F}_W/d^3r^3dr'^3 \right\rangle = \mathbf{R}\Big\{\big[c^2\rho\rho' - 2\mathbf{J}\cdot\mathbf{J}' + 3(\mathbf{R}\cdot\mathbf{J})(\mathbf{R}\cdot\mathbf{J}')/R^2 \\ + \omega\rho\mathbf{R}\cdot\mathbf{J}' - \omega\rho'\mathbf{R}\cdot\mathbf{J}\big] Q(R) - k^2\big[(\mathbf{R}\cdot\mathbf{J})(\mathbf{R}\cdot\mathbf{J}')/R^2\big] P(R)\Big\}, \end{aligned} \tag{21}$$

where ρ, ρ', $\mathbf{J}$, and $\mathbf{J}'$ refer here to only the space parts as given by Eqs.(14) and (15), and where

$$P(R) = (\cos kR)/R^3, \qquad Q(R) = (\cos kR + kR\sin kR)/R^3. \tag{22}$$

This time average result (21) is seen to satisfy Newton's third law; as it is directed along $\mathbf{R}$ and interchanging primes and unprimes yields only a change of sign, the functions P(R) and Q(R) being invariant to an interchange of primes and unprimes. Thus, considering the result of the previous Section 2.2 above, the Weber theory predicts a zero self torque on the Pappas-Vaughan Z-antenna.

2.4. Integrals for the Maxwell-Lorentz self torque on the Pappas-Vaughan Z-antenna

Substituting the time harmonic variations specified by Eqs.(13) into the first two of Eqs.(9) and (12) and taking a time average over a cycle, using Eqs.(20) the time average Maxwell-Lorentz force between volume elements becomes

$$2c^2 \left\langle d^6\mathbf{F}_M/d^3r d^3r' \right\rangle = \omega\rho\,\mathbf{J}'P(R) + \left[c^2\rho\rho'\mathbf{R} - \mathbf{R}(\mathbf{J}\cdot\mathbf{J}') + \mathbf{J}'(\mathbf{J}\cdot\mathbf{R})\right]Q(R), \tag{23}$$

where P(R) and Q(R) are defined by Eqs.(22) and ρ, ρ' **J**, and **J'** refer to the space parts as given by Eqs.(14) and (15). It may be seen that the second and third terms on the right of Eq.(23) satisfy Newton's third law. Considering Section 2.2 above, these terms will, thus, contribute nothing to the self torque on the Pappas-Vaughan Z-antenna. Only the first and fourth terms on the right of Eq.(23), violating Newton's third law, can contribute to a nonzero self torque.

It is convenient to write the self torque T as the sum of two terms: U the contribution from charge-current interactions given by the first term on the right of Eq.(23) and V the contribution from current-current interactions given by the fourth term on the right of Eq.(23); thus,

$$T = U + V, \tag{24}$$

where

$$U = (\omega/2c^2)\iint d^3r\,d^3r'\,\rho\,\mathbf{e}_z\cdot(\mathbf{s}\times\mathbf{J}')\,P(R),$$
$$V = (1/2c^2)\iint d^3r\,d^3r'\,\mathbf{e}_z\cdot(\mathbf{s}\times\mathbf{J}')\,Q(R), \tag{25}$$

where the ρ's and **J**'s are given by Eqs.(14) and (15), **s** is defined by Eq.(18), and P(R) and Q(R) are given by Eqs.(22).

The labor of evaluating the integrals in Eqs.(25) is considerably reduced by noting that the charge and current densities are all confined to the xy-plane; so **J'** has only x and y components; and the volume integrations reduce to integrations over x, y, x', and y'; thus,

$$U = (\omega/2c^2)\iint dx\,dy\,dx'\,dy'\,\rho\,(xJ'_y - yJ'_x)P(R),$$
$$V = (1/2c^2)\iint dx\,dy\,dx'\,dy'\,(xJ'_y - yJ'_x)\left[(x - x')J_x + (y - y')J_y\right]Q(R). \tag{26}$$

The integrals (26) may be broken down in terms of contributions form the various portions of the antenna. Thus U_{13} is the torque arising from charge-current interaction due to portion 3 acting on portion 1, U_{21} is the torque due to portion 1 acting on portion 2, etc. From symmetry it may be seen that the torque involving portion 3 equals the torque involving portion 1. In particular, to evaluate the torque on portion 3 a mathematical rotation of the antenna through 180° may be made to bring portion 3 into the original position of portion 1. Only the signs of the charges and currents are reversed as compared with the original situation before the rotation. But since the integrals involve a product of two currents or a product of a charge density and a current density; the integrals remain precisely the same as before the rotation. The torque involving portion 3 must, therefore, be identical to that produced by portion 1. It is sufficient to merely double the contributions involving portion 1 to obtain the net torque. The contributions due to charge-current and current-current interactions may then be written as

$$\begin{aligned} U &= 2U_{13} + 2U_{12} + 2U_{21} \\ V &= 2V_{13} + 2V_{12} + 2V_{21} \end{aligned} \tag{27}$$

Substituting Eqs.(14), (15), and (22) into the first of Eqs.(26) yields

$$\begin{aligned} 2U_{13} &= (\pi I^2/c^2)\int_0^{\lambda/4} dy \int_{-\lambda/4}^{0} dy' \sin ky \cos ky' (\cos kR'/R') \\ 2U_{12} &= (kI^2/c^2)\int_{-\lambda/2}^{\lambda/2} dx \int_0^{\lambda/4} dy\, y \sin ky \cos kx (\cos kR/R), \\ 2U_{21} &= (-kI^2/c^2)\int_{-\lambda/2}^{\lambda/2} dx \int_0^{\lambda/4} dy\, x \sin kx \cos ky (\cos kR/R), \end{aligned} \tag{28}$$

where here

$$R'^2 = \lambda^2 + (y - y')^2 \qquad \text{and} \qquad R^2 = (x - \lambda/2)^2 + y^2. \tag{29}$$

Similarly substituting Eqs.(14), (15), and (22) into the second of Eqs.(26) yields

$$2V_{13} = (I^2 \lambda/2c^2)\int_0^{\lambda/4} dy \int_{-\lambda/4}^{0} dy'\,(y - y')\cos ky \cos ky'\, Q(R'),$$

$$2V_{12} = (I^2/c^2)\int_{-\lambda/2}^{\lambda/2} dx \int_0^{\lambda/4} dy\, y^2 \cos kx \cos ky\, Q(R), \qquad (30)$$

$$2V_{21} = (-I^2/c^2)\int_{-\lambda/2}^{\lambda/2} dx \int_0^{\lambda/4} dy\, x(x - \lambda/2)\cos kx \cos ky\, Q(R),$$

where R and R' are given by Eqs.(29) and Q(R) is defined by the second of Eqs.(22).

2.5. Evaluation of the Maxwell-Lorentz integrals for the self torque on Pappas-Vaughan Z-antenna

The net Maxwell-Lorentz self torque on the Pappas-Vaughan Z-antenna given by Eq.(24) involves the integration of the integrals found in Eqs.(29) and (30). It may be noted from the definition of Q(R), Eq.(22), and Eqs.(29) that

$$(y - y')Q(R') = -\,\partial/\partial y\,(\cos kR'/R'),$$

$$y\,Q(R) = -\,\partial/\partial y\,(\cos kR/R), \qquad (31)$$

$$(x - \lambda/2)Q(R) = -\,\partial/\partial x\,(\cos kR/R).$$

Using this result (31), the integrals in Eqs.(30) may be integrated by parts yielding

$$2V_{13} = (I^2 \lambda/2c^2)\int_{-\lambda/4}^{0} dy'\cos ky' \left\{ -\cos ky\,(\cos kR'/R')\Big|_{y=0}^{y=\lambda/4} - k\int_0^{\lambda/4} dy\,\sin ky\,(\cos kR'/R') \right\}$$

(continued on next page)

$$2V_{12} = (I^2/c^2)\int_{-\lambda/2}^{\lambda/2} dx\cos kx\left\{-y\cos ky\,(\cos kR/R)\Big|_{y=0}^{y=\lambda/4}\right. \quad (32)$$

$$\left.+\int_0^{\lambda/4} dy\,(\cos ky - ky\sin ky)(\cos kR/R)\right\}$$

$$2V_{21} = (I^2/c^2)\int_0^{\lambda/4} dy\cos ky\left\{x\cos kx\,(\cos kR/R)\Big|_{x=-\lambda/2}^{x=\lambda/2}\right.$$

$$\left.-\int_{-\lambda/2}^{\lambda/2} dx\,(\cos kx - kx\sin kx)(\cos kR/R)\right\}.$$

It may be seen that the last term on the right of the first of Eqs.(32) equals $-2U_{13}$, as given by the first of Eq.(28); the last term on the right of the second of Eqs.(32) equals $-2U_{12}$, as given by the second of Eqs.(28); and the last term on the right of the third of Eqs.(32) equals $-2U_{21}$, as given by the third of Eqs.(28). Moreover, it may be seen that the second term on the right of the third of Eqs.(32) equals minus the second term on the right of the third of Eqs.(32). Combining terms for the total self torque using Eqs.(24), (27), (28), and (32), therefore, yields only the sum of the first terms appearing on the right of the three Eqs.(32), which involve only single integrations. Putting in the limits of integration arising from the first integrations and adding the resulting single integration terms in Eq.(30) yields the net time average Maxwell-Lorentz self torque on the Pappas-Vaughan Z-antenna as

$$T = (-I^2\lambda/2c^2)\int_0^{\lambda/4} dy\,(\cos^2 ky)/y. \quad (33)$$

This result (33), which arises primarily from the corners of the antenna, predicts an infinite Maxwell-Lorentz self torque on the antenna, there being a logarithmic singularity as $y \to 0$. The infinity arises from the fact that the current densities were assumed to be infinite, a finite current being confined to wires of infinitesimal cross sections. If wires are assumed to be of finite cross section and to be bent around small curves instead of sharp corners, the integral in Eq.(33) can be replaced by a rough realistic estimate by letting y = b, where b is a small nonzero parameter. In particular, for the Pappas-Vaughan setup b may be taken as roughly equal to λ/100 (which is about 1 cm for the antenna actually used). Thus,

$$\int_0^{\lambda/4} dy\cos^2 ky/y \rightarrow \int_{\lambda/100}^{\lambda/4} dy\cos^2 ky/y$$
$$\approx \int_{\lambda/100}^{\lambda/4} dy\,(1/2)/y = 1.610; \tag{34}$$

2.6. Discussion concerning the self torque on the Pappas-Vaughan Z-antenna

Pappas and Vaughan found from the power fed to their antenna of at least 35 watts and its impedance of 70 ohms that the peak current I was at least 1 ampere. Substituting this value of I and the wavelength λ = 2 m into Eq.(33), using Eq.(34), yields the estimated self torque on the Pappas-Vaughan Z-antenna as predicted by the Maxwell-Lorentz theory of at least

$$T \sim -0.805\, I^2\lambda/c^2 \sim -10^{-2}\ \text{Nt m}. \tag{35}$$

This is 5 orders of magnitude greater than the minimum torque of 10^{-7} Nt m that could have been observed. They observed no torque.

As an experimental check they had no difficulty in obtaining a strong deflection when a half-wavelength straight wire was brought into the neighborhood of one end of their antenna. The dipole induced in the wire by their antenna would be expected to produce an effect of the same order of magnitude, but smaller, than that predicted by the Maxwell-Lorentz theory.

It is concluded that the nonzero self torque predicted by the Maxwell-Lorentz theory does not agree at all with the experimental result of Pappas and Vaughan; while the zero torque predicted by the Weber field theory and Newton's third law does agree with their result to within the limits of the sensitivity of their setup.

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