based on statistical reasoning, can be such failures. The concept of evolution lies in the same province. It is not a theory but an axiom just as much as were Kepler's laws axioms. However, as the latter became theorems in a more general theory so this concept can become a theory in a more generalized, non-standard science. Here the second law of thermodynamics does not hold but causality does.

Conclusion

These, and other, speculations are all very well but for realization the link between matrices and elliptic functions has to be forged in detail. So far it is only a case of a recipe in search of a chef, or better, a *cordon bleu*.

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Weber electrodynamics extended to include radiation

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Abstract Weber's law for the force between moving charges satisfies Newton's Third Law, the conservation of energy, and yields Ampere's original empirical law for the force between current elements. Weber's law predicts the observed force between two closed current loops. It yields the observed temporal and motional electromagnetic induction, and the observed force on Ampere's bridge. The electromagnetic field appropriate for the Weber force law is derived. This Weber field is then extended to rapidly varying effects and radiation by introducing time retardation. An additional electromagnetic wave, the Weber wave is obtained. Absolute space is included by introducing time retardation using the phase velocity observed in a moving system as predicted by the Voigt–Doppler effect.

Introduction

The electromagnetic theory of Weber,[1] as first presented in 1846, is based on the force between two moving point charges using the separation distance, the relative velocity, and the relative acceleration. His theory is the only electromagnetic theory ever proposed that satisfies both Newton's Third Law and the conservation of energy. Weber's original theory was limited to slowly varying effects where time retardation and radiation were not involved. It is an 'action-at-a-distance' theory, no fields being necessary. It is now known that electromagnetic waves exist and that a field theory is required to describe such waves. This paper, therefore, first expresses Weber's original action-at-a-distance theory in terms of fields and then second introduces time retardation to yield electromagnetic waves. In this way, it is possible to obtain an electromagnetic theory compatible with all of the evidence which is consistent with Newton's Third Law and the conservation of energy.

Maxwell's theory, being based upon the Biot–Savart Law, does not satisfy Newton's Third Law. Maxwell's theory cannot, therefore, predict the observed force on Ampere's bridge. Because Maxwell's radiation theory[2] is a generalization from erroneously assumed slowly varying effects it fails to predict the correct radiation field. An additional wave, the 'Weber wave', as derived here, is needed.

Weber assumed that the energy U of two interacting charges q_2 and q_1 at positions $\mathbf{r}_2$ and $\mathbf{r}_1$ moving with the velocities $\mathbf{v}_2$ and $\mathbf{v}_1$ is

$$U = q_1 q_1[1/r - (\mathbf{v}\cdot\mathbf{r})^2/2c^2r^3] \quad (1)$$

where c is the velocity of light and

$$\mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1, \quad \mathbf{v} = \mathbf{v}_2 - \mathbf{v}_1 \quad (2)$$

The first term on the right of equation (1) is simply the Coulomb potential energy.

When the system changes spontaneously with time this energy U changes at the rate

$$dU/dt = -(q_2 q_1 \mathbf{v}\cdot\mathbf{r}/r^3)[1 + v^2/c^2 - 3(\mathbf{v}\cdot\mathbf{r})^2/2c^2r^2 + (\mathbf{r}\cdot d\mathbf{v}/dt)/c^2] \quad (3)$$

where for $\mathbf{v}\cdot\mathbf{r} > 0$ the charges are receding from each other and U decreases with time (assuming $v/c \leq 1$). Because this result (3) may be written in the form

$$dU/dt = -\mathbf{v}\cdot\mathbf{F}_w \quad (4)$$

the Weber force on charge q_2 due to charge q_1 is given by

$$\mathbf{F}_w = (q_2 q_1 \mathbf{r}/r^3)[1 + v^2/c^2 - 3(\mathbf{v}\cdot\mathbf{r})^2/2c^2r^2 + (\mathbf{r}\cdot d\mathbf{v}/dt)/c^2] \quad (5)$$

This result (5) satisfies Newton's Third Law; since the force is along $\mathbf{r}$; and, interchanging subscripts $_1$ and $_2$ (to obtain the force on charge q_1 due to q_2), changes the sign of $\mathbf{F}_w$, the magnitude remaining the same.

The Weber force also conserves energy. Assuming that the charges also have mass, multiplying Newton's Second Law for particle 2 by $\mathbf{v}_2$ and Newton's Second Law for particle 1 by $\mathbf{v}_1$ and adding yields

$$\mathbf{v}_2\cdot d\mathbf{p}_2/dt + \mathbf{v}_1\cdot d\mathbf{p}_1/dt = (\mathbf{v}_2 - \mathbf{v}_1)\cdot\mathbf{F}_w = -dU/dt \quad (6)$$

where $\mathbf{p}_2$ and $\mathbf{p}_1$ are the momenta of the particles. This result (6) is immediately integrable. The total energy $U + T$, where T is the kinetic energy of the system, then depends only upon the end points. The total energy is independent of the path or the rate of change of the system between the end points; thus, energy is conserved.

Ampere's Law and the absurdity of the Biot–Savart Law

In order to make it appear that Ampere's original empirical law[3] is satisfied the name *Ampere's Law* is often usurped for force laws that are not compatible with Ampere's original law. For example, many text books label the Biot–Savart Law as *Ampere's Law*. Ampere's original empirical law for the force $\mathbf{F}_A$ on a current element $I_2 d\mathbf{s}_2$ at $\mathbf{r}_2$ due to a current element $I_1 d\mathbf{s}_1$ at $\mathbf{r}_1$ separated by the distance $\mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1$ is

$$c^2\mathbf{F}_A = I_2 I_1 \mathbf{r}[-2d\mathbf{s}_2\cdot d\mathbf{s}_1/r^3 + 3(d\mathbf{s}_2\cdot\mathbf{r})(d\mathbf{s}_1\cdot\mathbf{r})/r^5] \quad (7)$$

Ampere's Law (7) clearly satisfies Newton's Third Law: The force is directed

along $\mathbf{r}$, and it changes sign when the subscripts $_1$ and $_2$ are interchanged to give the force on current element $I_1 d\mathbf{s}_1$ due to $I_2 d\mathbf{s}_2$. The Biot–Savart Law is quite different; it is

$$c^2\mathbf{F}_B = I_2 I_1 d\mathbf{s}_2 \times (d\mathbf{s}_1 \times \mathbf{r})/r^3 = I_2 I_1[-(d\mathbf{s}_2\cdot d\mathbf{s}_1)\mathbf{r} + d\mathbf{s}_1(d\mathbf{s}_2\cdot\mathbf{r})]/r^3 \quad (8)$$

The Biot–Savart Law (8) clearly does *not* satisfy Newton's Third Law: the force is not directed along $\mathbf{r}$, and interchanging the subscripts $_1$ and $_2$ does *not* yield the negative for the force on $I_1 d\mathbf{s}_1$ due to $I_2 d\mathbf{s}_2$. It may be noted that these laws (7) and (8) refer to moving charges by using the replacements

$$q_1\mathbf{v}_1 = I_1 d\mathbf{s}_1, \quad q_2\mathbf{v}_2 = I_2 d\mathbf{s}_2 \quad (9)$$

The absurdity of the Biot–Savart Law (or any law violating Newton's Third Law) can be immediately demonstrated. The self force on a closed current loop[4] may be considered. Ampere's Law (7), which satisfies Newton's Third Law, clearly yields zero for the self force on a closed current loop. Mechanically coupling current element $I_2 ds_2$ together with current element $I_1 ds_1$, the Biot–Savart Law (8) predicts a net nonvanishing force on the two coupled current elements equal to

$$c^2\mathbf{F}_B = I_2 I_1 \mathbf{r} \times (d\mathbf{s}_1 \times d\mathbf{s}_2)/r^3 \quad (10)$$

Integrating this result (10) with respect to ds_2 over a portion 2 of a closed current loop and with respect to ds_1 over the remaining portion 1 of the loop yields a net force on the entire closed current loop equal to

$$c^2\mathbf{F}_B = I^2 \int_1 \int_2 [\mathbf{r} \times (d\mathbf{s}_1 \times d\mathbf{s}_2)]/r^3 \quad (11)$$

which need not vanish. By altering the way in which the loop is divided into portions 1 and 2, which is merely a matter of labelling, the force predicted by equation (11) can be altered. Nothing need be changed physically. Interchanging labels 1 and 2 reverses the direction of the force. This net force on a closed current loop could be used in principle to lift a space ship by its own *boot straps* from the forces inside the space ship itself. And no energy has to be expended. The absurdity is clear.

It is frequently claimed that only the net ponderomotive force between two closed current loops can be observed to test proposed laws for the force between current elements or moving charges. Since the Biot–Savart Law (equation (8)) and the Ampere Law (equation (7)) yield precisely the same net force between two closed current loops; it is frequently claimed that it does not matter which law is used. To try to argue from an obviously false premise, the Biot–Savart Law violating Newton's Third Law, in order to obtain a correct conclusion is hardly satisfactory scientific thinking. Moreover, observations are *not* limited to the force between two closed current loops. Three independent observations not limited to the force between two closed current loops are readily available to test the validity of any proposed force law between current elements or moving charges: (1) the force on one portion of a closed current loop due to the remaining portion of the loop can be measured—the Ampere bridge experiment.[3] (2) The force on a charge moving in a vacuum in the presence of an external closed current loop can be observed. (3) Müller (personal communication) has shown that the induced electric field in a conductor moving in the presence of an external closed current loop is localized. The induced field is not distributed uniformly around the whole metalic loop as required by the erroneous Faraday–Maxwell theory. Müller's observations can, thus, be used to determine the force on a single current element due to an external closed current loop (as also revealed by observations of type (2) above). In principle, one could also measure the force between moving charges directly in a vacuum; but the Coulomb force would probably be too large to detect the much smaller velocity effects.

Force on Ampere's bridge

A straightforward integration of the Ampere Law (7) for the force on Ampere's bridge[3] yields a result compatible with the experimental observations made over the past 160 years.[3,5–8] A similar straightforward integration of the Biot–Savart Law (8) (see below) does not yield a result compatible with either the experimental observations nor with self consistency.

For some strange reason it is frequently claimed that the Ampere Law (7) is wrong and the Biot–Savart Law (8) is correct. The experimentally established strong repulsive force between colinear current elements, as given by the Ampere Law (7) is, thus, frequently ignored or denied. The acceptance of Maxwell's electrodynamics, which is based upon the Biot–Savart Law (8) and which rejects the Ampere Law (7) has resulted in very expensive errors in attempts to achieve thermonuclear fusion and in the design of high-energy particle accelerators.

The problem of calculating the force on Ampere's bridge directly from either the Ampere formula (7) or the Biot–Savart formula (8) has met with mathematical difficulties. Cleveland[6] attempted to use linear current elements, or wires of vanishing cross-section. When the separation distance between such linear current elements goes to zero the contribution to the force, according to equation (7), then becomes infinite. Cleveland, therefore, introduced a small arbitrary finite separation to obtain a finite result. Unfortunately the effect he predicted then became primarily dependent upon his arbitrary separation distance. Robertson,[9] also using linear current elements, terminated his integrations in terms of an arbitrary diameter of the wire he was considering in order to obtain a finite result. (Both Cleveland and Richardson also made errors in sign (not the same error) in their final formulas.) Graneau[8] has also used linear current elements. To avoid infinites he has arbitrarily chosen a smallest interval for his computer calculations. Because no arbitrary parameter can be involved in the physics; these theories fail to provide adequate quantitative predictions.

The only correct theoretical procedure involves going simply to three dimensions with three-dimensional current elements. Using three-dimensional current densities no infinities can arise. Contributions to integrals remain finite when the separation distance between volume current elements is allowed to go to zero.

To obtain an exact result in closed mathematical form without any artificial singularities it is convenient to turn to laminar geometry, as indicated in Figure 1. The laminar thickness τ perpendicular to the page in the z direction is not shown. The net force on Ampere's bridge is then obtained by integrating

$$d^6\mathbf{F}_A/d^3\mathbf{r}_2 d^3 r_1 = \mathbf{r}[-2\mathbf{J}_2\cdot\mathbf{J}_1/r^3 + 3(\mathbf{J}_2\cdot\mathbf{r})(\mathbf{J}_1\cdot\mathbf{r})/r^5] \tag{12}$$

where $\mathbf{J}$ is the volume current density. The case of a thin lamina (τ small) of narrow width (w small), where τ and w can be neglected in comparison to the other dimensions of the circuit, is considered. The Biot–Savart Law (8), involving no possible singularities, can be immediately converted to surface integrals ($\tau = 0$) involving surface currents $\mathbf{K}$, where $K = I/w$; thus, when A_2 is the surface area of the bridge and A_1 is the surface area of the remainder of the circuit,

$$c^2\mathbf{F}_B = \int\int_{A_2} da_2 \int\int_{A_1} da_1[-(\mathbf{K}_2\cdot\mathbf{K}_1)\mathbf{r} + \mathbf{K}_1(\mathbf{K}_2\cdot\mathbf{r})]/r^3 \tag{13}$$

The surface areas A_2 and A_1 may be conveniently broken up into six portions, as indicated in Figure 1. The force on Ampere's bridge may then be calculated

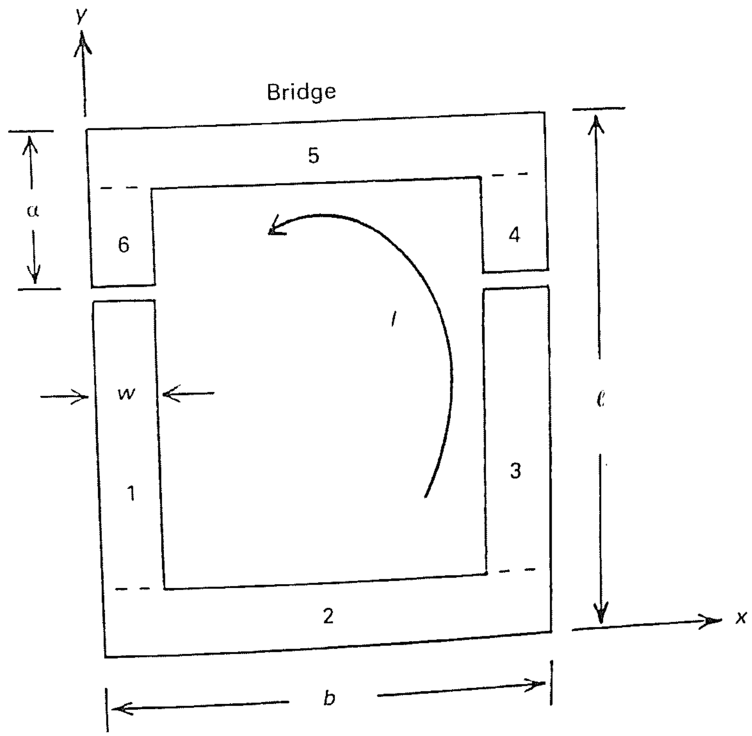

Figure 1 *Ampere's bridge on which the force due to the remainder of the circuit is calculated directly using the Ampere original force law (7) and the Biot–Savart Law (8).*

as the sum of nine integrals. Symmetry considerations reduce the labour involved. For example, the integral for the force on portion 6 due to portion 1 is

$$c^2F_A(6,1)/I^2 = \tag{14}$$

$$(1/w^2\tau^2)\int_0^w dx_2 \int_{\ell-\alpha}^{\ell-w} dy_2 \int_0^\tau dz_2 \int_0^w dx_1 \int_w^{\ell-\alpha} dy_1 \int_0^\tau dz_1\,(-2Y/r^3 + 3Y^3/r^5)$$

$$= 13/12 - \pi/3 + (2/3)\ ln 2 + ln(\alpha/\ell) + ln[(\ell-\alpha)/w]$$

where $Y = y_2 - y_1$ and $r^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2$ and τ has been set equal to w (wire of square cross-section) and where w has been neglected compared with other dimensions of the circuit. In lengthy, but straightforward analysis, the force on the bridge according to Ampere's Law (7) is found to be in the positive y direction and equal to

$$F_A = 2(I^2/c^2)[13/12 - \pi/3 - (2/3)ln 2 + \sqrt{1 + b^2/\ell^2} - ln(1 + \sqrt{1 + b^2/\ell^2}) + ln(b/w)] \tag{15}$$

for w small compared with ℓ, b, α, or $\ell - \alpha$. If the width of the bridge b may be neglected as compared with ℓ, equation (15) yields

$$F_A = 2(I^2/c^2)[-0.1191\ldots + ln(b/w)] \tag{16}$$

Note that the large term involving $ln\ w$ in equations (15) or (16) arises from the interaction of the parallel portions of the loop near the contact, i.e., the force on 6 due to 1 and the force on 4 due to 3 (as shown in Figure 1). The result (16), thus, depends primarily upon the repulsive force between colinear current elements as given by Ampere's Law (7).

Carrying out a similar straightforward integration of the Biot–Savart Law (8), the force on the bridge is supposed to be in the y direction and equal to

$$F_B = 2(I^2/c^2)[-1 + \sqrt{1 + b^2/\ell^2} - ln(1 + \sqrt{1 + b^2/\ell^2}) + ln(1 + \sqrt{1 + b^2/\alpha^2})], \tag{17}$$

which for b small compared with ℓ or α is simply

$$F_B = 0. \tag{18}$$

This Biot–Savart result (17) or (18) is radically different from the directly calculated Ampere result (15) or (16). The strong repulsive force observed[3,6–9] is in agreement with the Ampere result. The experimental observations do not agree with the weak or zero force predicted by the Biot–Savart Law.

Actually the Biot–Savart result (17) is *absurd*; as the force on the remainder of the circuit does not equal the negative of this result. The force on the remainder of the circuit may be readily obtained by simply changing the sign of equation (17) and replacing α by $\ell - \alpha$. When the two portions of the closed circuit are mechanically coupled together there is then supposed to be a net nonvanishing force on the loop given by

$$F_B = 2(I^2/c^2)[ln(1 + \sqrt{1 + b^2/\alpha^2}) - ln(1 + \sqrt{1 + b^2/(\ell - \alpha)^2})] \tag{19}$$

In principle, this force could be used to lift a space ship or drive an automobile without any external forces or without energy expenditure. This result (equation (19)) is merely a concrete example of the absurdity already given by equation (11).

Cleveland[6] actually measured the forces on the two portions of the closed circuit. He found the forces were, in fact, equal and oppositely directed as required by Newton's third law. Cleveland, thus, disproved the Biot–Savart law directly experimentally.

Weber force yields Ampere's law

Weber's force law (5) yields Ampere's empirical force law (7). Four forces act on an element of a conductor ds_2 carrying a current I_2 due to an element ds_1 of another conductor carrying a current I_1:

(1) the force between positive stationary ions ($+q_2$, $\mathbf{v}_2 = 0$; $+q_1$, $\mathbf{v}_1 = 0$),
(2) the force between the stationary positive ions in ds_2 and the moving electrons in ds_1 ($+q_2$, $\mathbf{v}_2 = 0$; $-q_1$, $-\mathbf{v}_1$),
(3) the force between the moving electrons in ds_2 and the stationary positive ions in ds_1 ($-q_2$, $-\mathbf{v}_2$; $+q_1$, $\mathbf{v}_1 = 0$), and
(4) the force between the moving electrons in ds_2 and the moving electrons in ds_1 ($-q_2$, $-\mathbf{v}_2$; $-q_1$, $-\mathbf{v}_1$).

Substituting the appropriate values indicated in the brackets into Weber's Law (5) and adding readily yields the net ponderomotive force on ds_2 carrying a current I_2 due to an element ds_1 carrying a current I_1 in agreement with Ampere's empirical law (7). The Coulomb terms and the acceleration terms appearing in (5) cancel out in this case.

Weber field for electrostatics and magnetostatics

It is clear that the electrostatic potential to be associated with the first term of the Weber force (5), the Coulomb force, is given by the usual expression,

$$\Phi(\mathbf{r}_2) = \iiint_1 d^3r_1\, \varrho_1(\mathbf{r}_1)/r \tag{20}$$

where $\varrho_1(\mathbf{r}_1)$ is the charge density distribution of the source. The Coulomb force per unit volume charge density $\varrho_2(\mathbf{r}_2)$ is then given by

$$f_W(\text{electrostatic}) = -\varrho_2\nabla_2\Phi \tag{21}$$

For the magnetostatic case, where fields and forces are associated with steady currents in conductors, the Ampere force law (7) is the appropriate form of Weber's force law (5). In this case, the force per unit volume on a current density $\mathbf{J}_2$ due to a distribution of current density $\mathbf{J}_1$ may be written from equation (7) in the form

$$c^2\mathbf{f}_W(\text{magnetostatic}) = -\iiint_1 \mathbf{r}[\mathbf{J}_2\cdot\mathbf{J}_1/r^3 + (\mathbf{J}_2\cdot\nabla_2)(\mathbf{J}_1\cdot\nabla_1)(1/r)]d^3r_1 \tag{22}$$

In order to remove $\mathbf{J}_2(\mathbf{r}_2)$ from the integral the following identities may be considered:

$$\mathbf{r}\mathbf{J}_1\cdot\nabla_2(1/r) = -\mathbf{r}(\mathbf{r}\cdot\mathbf{J}_1)/r^3 = \mathbf{r}\cdot\mathbf{J}_1\nabla_2(1/r) = \nabla_2[(\mathbf{J}_1\cdot\mathbf{r})/r] - \mathbf{J}_1/r \tag{23}$$

Using equation (23) the right-hand side of equation (22) becomes

$$\mathbf{J}_2\times(\nabla_2\times\int\mathbf{J}_1/r) - \mathbf{J}_2\nabla_2\cdot\int\mathbf{J}_1/r + (\mathbf{J}_2\cdot\nabla_2)\nabla_2\int(\mathbf{r}\cdot\mathbf{J}_1)/r \tag{24}$$

where the notation for the volume integrations has been abbreviated.

Introducing the usual vector potential $\mathbf{A}$ and a magnetic scalar potential Γ, the Weber magnetostatic potential field becomes

$$c\mathbf{A} = \iiint_1 d^3r_1\mathbf{J}_1/r, \qquad c\Gamma = \iiint_1 d^3r_1\mathbf{J}_1\cdot\mathbf{r}/r \tag{25}$$

In terms of these potential fields, $\mathbf{A}$ and Γ the force per unit volume on the current density $\mathbf{J}_2$ due to the steady current distribution $\mathbf{J}_1$ is

$$c\mathbf{f}_W(\text{magnetostatic}) = \mathbf{J}\times(\nabla\times\mathbf{A}) - \mathbf{J}\nabla\cdot\mathbf{A} + (\mathbf{J}\cdot\nabla)\nabla\Gamma \tag{26}$$

where the subscripts $_2$ have been dropped.

For the special limiting case when $\nabla\cdot\mathbf{A} = \Gamma = 0$ this result (26) reduces to the Maxwell theory. It may be shown that this limiting case involves only contained closed current loop sources. In general this need not be true. For example, to predict the force on Ampere's bridge the source must be taken as the portion of circuit apart from the bridge where the current does not form closed loops and is not contained.

Weber force on a moving charge due to a current-carrying wire

A moving charge q_2, taken as positive, can interact with the positive stationary ions in a wire where the force involves: $(+q_2, +\mathbf{v}_2; +q_1, \mathbf{v}_1 = 0)$; and it can interact with the negative moving electrons where the force involves: $(+q_2, \mathbf{v}_2; -q_1, -\mathbf{v}_1)$. Adding these two forces, using equation (5), yields the Weber force on a moving charge q_2 due to a current carrying conductor as

$$c^2\mathbf{F}_W = q_2q_1\mathbf{r}[-2\mathbf{v}_2\cdot\mathbf{v}_1/r^3 + 3(\mathbf{v}_2\cdot\mathbf{r})(\mathbf{v}_1\cdot\mathbf{r})/r - (\mathbf{r}\cdot d\mathbf{v}_1/dt)/r^3 - v_1^2/r^3 + 3(\mathbf{v}_1\cdot\mathbf{r})^2/2r^5] \tag{27}$$

The last two terms in the square brackets represent the force on a stationary charge produced by a current-carrying wire. It will be shown below that this force is very small and may be neglected.

To obtain the field associated with this Weber force on a moving charge due to a current-carrying wire it is only necessary to examine the acceleration term. The other terms can be represented as in the magnetostatic case, as given by equations (25) and (26), where $\mathbf{J}$ is replaced by $q_2\mathbf{v}_2$. The force on charge q_2 due to a time rate of change of the source current J_1 is given from equation (27) by

$$-q_2\iiint_1 d^3r_1\mathbf{r}(\mathbf{r}\cdot\partial\mathbf{J}_1/\partial t)/r^3 = \tag{28}$$

$$q_2\partial/\partial t\left[\nabla_2\iiint_1 d^3r_1(\mathbf{r}\cdot\mathbf{J}_1)/r - \iiint_1 d^3r_1\mathbf{J}_1/r\right] =$$

$$q_2(\nabla_2\partial\Gamma/\partial t - \partial\mathbf{A}/\partial t)$$

Combining equations (21), (26), and (28) then yields the Weber force on a charge q moving with the velocity $\mathbf{v}$ in a potential field Φ, $\mathbf{A}$, and Γ, defined by equations (20) and (25),

$$c\mathbf{F}_W = q[-c\nabla\Phi - \partial\mathbf{A}/\partial t + \partial\Gamma/\partial t + \mathbf{v}\times(\nabla\times\mathbf{A}) - \mathbf{v}\nabla\cdot\mathbf{A} + (\mathbf{v}\cdot\nabla)\nabla\Gamma] \tag{29}$$

For the limiting Maxwell case, where only confined current loop sources are involved and $\nabla\cdot A = \Gamma = 0$, The Weber force (29) reduces to the Lorentz force.

Weber induction for stationary circuits

Induction may be defined as the electromagnetic force on the electrons (the carriers, counted as positive by convention) in a conductor. The induced force may be contrasted with the ponderomotive force that acts on the metal as a whole (or essentially on the fixed positive ions). The induced force on the electrons in a metal can produce a current or a charge separation. The force on the positive ions can produce neither current nor charge separation. The Weber force on the electrons produced by another current-carrying wire is the same as the force on a charge as already given by equation (27) or for distributed charge and current sources by equation (29), where by convention the carriers (the electrons) are counted as positive.

Since electrons are not on average accelerated in a metal the forces induced on the electrons must be balanced by other forces. When electrons flow the induced force is balanced by an Ohmic drag. When the induced force on the electrons can result in no current flow, then a charge separation must produce an electrostatic field that balances the induced force.

In this case, currents do not change with time and the conductors are stationary. The Hall effect arises when the direction of the induced force on the electrons is perpendicular to the flow of electrons. In this direction, no Ohmic drag can balance the induced force. In this case, a charge separation must occur in the direction of the induced field to balance the induced field. Considering the effect on a current density instead of on a single moving charge, q can be replaced by ϱ and $q\mathbf{v}$ by $\mathbf{J}$ to yield the force per unit volume $\mathbf{f}$. The Hall effect may then be obtained from equation (29) by neglecting Φ and the time variations and by considering only the force perpendicular to $\mathbf{J}$. The induced Hall force according to the Weber theory is then

$$c\mathbf{f}_W(\text{Hall}) = \mathbf{J} \times (\nabla \times \mathbf{A}) + [(\mathbf{J}\cdot\nabla)\nabla\Gamma]_\perp \tag{30}$$

where the subscript $\perp$ means the component perpendicular to $\mathbf{J}$.

Faraday induction

Faraday was interested in the induced force that could produce current in a closed stationary metalic loop. The force of interest is then parallel to the current. From equation (29), neglecting the electrostatic potential, this induced force is

$$c\mathbf{f}_W(\text{Faraday}) = [\varrho\partial(-\mathbf{A} + \nabla\Gamma)/\partial t + (\mathbf{J}\cdot\nabla)\nabla\Gamma]_{\parallel} - \mathbf{J}\nabla\cdot\mathbf{A} \tag{31}$$

where the subscript $\parallel$ means the component parallel to $\mathbf{J}$. Faraday was interested only in the net effect induced in a closed loop for the special limiting case where the inducing fields arise only from contained closed current loop sources where $\nabla\cdot\mathbf{A} = \Gamma = 0$. In this limiting case integrating equation (31) about a closed loop, the net electromotive (EMF) force becomes

$$\text{EMF} = \oint \mathbf{F}_W(\text{Faraday})\cdot d\mathbf{s}/q_2 = -\oint d\mathbf{s}\cdot(\partial\mathbf{A}/\partial t) = \partial\Phi/\partial t \tag{32}$$

where Φ is the magnetic flux through the loop. This Faraday result (32) is misleading as: (1) only confined closed current loop sources are assumed to induce the effect. And (2) the integration around a closed current loop averages out any possible variation in the induced force around the closed loop. The Faraday result (32) cannot, therefore, represent the general detailed result provided by Weber's formula (31). Note also that, in general, a magnetic flux cannot even be defined; since closed current loops need not always be involved. Müller's experiments (personal communication) demonstrate this localization of induction.

Weber induction for moving circuits

To obtain the induced force on electrons in a moving conductor due to another moving current carrying conductor Weber's force law (5) may be considered

for velocities of the electrons written as the sum of the velocity relative to the conductor $\mathbf{v}$ and the velocity of the conductor itself $\mathbf{v}'$. The force on a positive charge $+q_2$ moving with the velocity $\mathbf{v}_2 + \mathbf{v}_2'$ is then the sum of the force given by the positive ions $+q_1$ in the source moving with the velocity $\mathbf{v}_1'$ and the force given by the negative electrons $-q_1$, moving with the velocity $-\mathbf{v}_1 + \mathbf{v}_1'$. Using equation (5) and summing the two forces yields

$$\begin{aligned} c^2\mathbf{F}_W = q_2 q_1 \mathbf{r}\,\{&-2\mathbf{v}_1\cdot(\mathbf{v}_2 + \mathbf{v}_2')/r^3 + 3(\mathbf{v}_1\cdot\mathbf{r})\,[(\mathbf{v}_2 + \mathbf{v}_2')\cdot\mathbf{r}]/r^5 \\ &- (\mathbf{r}\cdot d\mathbf{v}_1/dt)/r^3 - (\mathbf{v}_1 - \mathbf{v}_1')^2/r^3 + \mathbf{v}_1'^2/r^3 \\ &- 3(\mathbf{v}_1'\cdot\mathbf{r})^2/2r + 3[(\mathbf{v}_1 - \mathbf{v}_1')\cdot\mathbf{r}]^2/2r^5\} \end{aligned} \tag{33}$$

The last four terms involving $(\mathbf{v}_1 - \mathbf{v}_1')^2$ and $\mathbf{v}_1'^2$ are again so small that they may be neglected, as will be shown below.

The surprising feature of this result (33) is that the velocity of the source conductor $\mathbf{v}'$ does not enter in, the effect of the motion of the positive ions cancelling out the effect of the corresponding motion of the negative electrons. Note that the accelerations of the two conductors produces no effect, the action on positive and negative charges cancelling each other in this case.

In a moving conductor in the field of a stationary source

Comparing equations (33) and (27) (neglecting terms in v_1^2 and $v_1'^2$), the induction is seen to be a sum of two terms, one involving the velocity of the electrons (taken as positive) relative to the conductor $\mathbf{v}_2$ and the other involving the velocity of the conductor itself $\mathbf{v}_2'$. The induction involving the relative velocity $\mathbf{v}_2$ has already been discussed so that only the additional effect due to the motion of the conductor itself need be considered here. The induction due to the motion of the conductor is

$$c_2\mathbf{F}_W(\text{in moving conductor}) = q_2 q_1 \mathbf{r}[-2\mathbf{v}_1\cdot\mathbf{v}_2'/r^3 + 3(\mathbf{v}_1\cdot\mathbf{r})(\mathbf{v}_2'\cdot\mathbf{r})/r^5] \tag{34}$$

For a distributed current source the induced force on the electrons (counted as positive) in a moving conductor is then

$$c\mathbf{f}_W(\text{in moving conductor}) = \varrho[\mathbf{v}_2' \times (\nabla \times \mathbf{A}) - \mathbf{v}_2'\nabla\cdot\mathbf{A} + (\mathbf{v}_2'\cdot\nabla)\nabla\Gamma] \tag{35}$$

following the analysis leading to equation (26). As for induced effects, in general, this result (35) can yield two possible effects: (1) an electromotive force given by integrating in the direction of the current around a closed loop, and (2) a charge displacement and an electrostatic field transverse to the direction of current flow.

In a stationary conductor in the field of a moving source

It might at first be thought that, because equation (33) contains no term involving the motion of the source, there should be no induction when the source is moved. Strictly speaking this is true for the effect between two point charges; but extended sources produce an additional effect. In particular, potentials are defined in terms of coordinates fixed relative to the source. When the source moves the stationary observer then sees an additional apparent time change due to this motion; thus,

$$d\mathbf{A}/dt = (\mathbf{v}_1'\cdot\nabla)\mathbf{A} + \partial\mathbf{A}/\partial t, \qquad d\Gamma/dt = (\mathbf{v}_1'\cdot\nabla)\Gamma + \partial\Gamma/\partial t \tag{36}$$

The additional induced effect not considered above is then

$$c\mathbf{f}_W \text{ (due to moving source)} = -\varrho(\mathbf{v}_1'\cdot\nabla)(\mathbf{A} - \nabla\Gamma) \tag{37}$$

For point charges this induced force (37) vanishes. It also vanishes if there is no variation in $\mathbf{A}$ or Γ along the direction of $\mathbf{v}_1'$. Thus, for example, if current carrying wires are moved parallel to themselves $\mathbf{A}$ or Γ cannot change and no induction can occur. The case of unipolar induction, where $\mathbf{v}_1'\cdot\nabla\mathbf{A} = 0$, has been carefully experimentally investigated by Kennard[10] and Müller (personal communication).

Weber force on a static charge due to a current

The Weber force (5) says that a static charge q_2 will experience a force due to a wire carrying current. Adding the two forces involving ($+q_2$, $\mathbf{v}_2 = 0$; $+q_1$, $\mathbf{v}_1 =$ 0) and ($+q_2$, $\mathbf{v}_2 = 0$; $-q_1$, $-\mathbf{v}_1$) according to equation (5), where $d\mathbf{v}_1/dt = 0$, yields

$$c^2\mathbf{F}_W = q_2q_1\mathbf{r}[-v_1^2/r^3 + 3(\mathbf{v}_1\cdot\mathbf{r})^2/2r^5] \tag{38}$$

This force is very small and has been neglected above.

To demonstrate just how small this force is, a specific numerical example may be considered where the force on a positive charge q_2 separated a distance b from an infinitely long straight wire with a current I is calculated. Let q_1 be the charge of the conduction electrons per unit length in the wire, $q_1\mathbf{v}_1 = I d\mathbf{s}_1$. The force on the charge q_2, which is perpendicular to the wire, is obtained from equation (38) by integrating along the wire (in the y direction); thus,

$$F_W = (q_2Iv_1b/c^2)\int_0^\infty dy(-2/r^3 + 3y^2/r^5) = -q_2Iv_1/c^2b \tag{39}$$

For the particular values $q_2 = 10^{-10}$ C, $I = 10^3$ A, $v_1 = 10$ cm/s, and $b = 1$ cm the force is $F_W = 10^{-8}$ dyne, a truly negligible effect.

The velocity squared terms involving v_1^2 are always negligible, as these terms arise from a very small apparent decrease in the charge of the electrons due to their motion. An effective positive charge may, thus, be assumed that is proportional to $q_1' = q_1v_1^2/c^2$. For a current-carrying wire the effective charge per unit length is then approximately

$$dq_1'/ds_1 = Iv_1/c^2 = I'v_1/10c$$

where I' is measured in amperes. Since the drift velocity v_1 of the conduction electrons in a metal is only of the order of centimetres per second; the effective charge distribution is extremely small. The effect on a static charge distribution due to the v_1^2 terms in equations (38), (27) and (33) may, thus, always be neglected.

Weber theory extended to include radiation

For electrostatics, magnetostatics, and slowly time-varying fields the Weber theory fits all of the observed facts as shown above. It is now only necessary to extend the Weber theory to rapidly varying effects and electromagnetic radiation. This can be done by simply generalizing the potential field Φ, $\mathbf{A}$, and Γ, equations (20) and (25), to include time retardation. The fields observed are assumed to arise from a state of the source at an earlier time, the retarded time t^*, as given by

$$t^* = t - r/c \tag{40}$$

where $r = |\mathbf{r}_2 - \mathbf{r}_1|$ is the distance between source and observer. The extended Weber field is then defined by

$$\Phi = \int\int\int d^3r_1\varrho_1(\mathbf{r}_1, t - r/c)/r$$

$$\mathbf{A} = \int\int\int d^3r_1\mathbf{J}_1(\mathbf{r}_1, t - r/c)/r \tag{41}$$

$$\Gamma = \int\int\int d^3r_1\mathbf{r}\cdot\mathbf{J}_1(\mathbf{r}_1, t - r/c)/r$$

where the force on a charge moving in this field is given by equation (29). The extended Weber electrodynamics is then prescribed completely by equations (41) and (29).

The integral expressions (41) can also be expressed as differential equations; thus,

$$\Box^2\Phi = -4\pi\varrho, \qquad \Box^2\mathbf{A} = -4\pi\mathbf{J}/c$$

$$[\nabla^4 - \partial^4/\partial(ct)^4]\,\Gamma = 8\pi\partial\varrho/\partial ct \tag{42}$$

where $\Box^2 = \nabla^2 - \partial^2/\partial(ct)^2$, and $\nabla^2\mathbf{A} = -\nabla\times\nabla\times\mathbf{A} + \nabla\nabla\cdot\mathbf{A}$

Since Γ does not, in general, vanish for a finite source; the third of equations (41) or the third of equations (42) yields an additional electromagnetic wave, the 'Weber wave', not predicted by the Maxwell theory. As the Γ field must exist to satisfy Newton's Third Law and the conservation of energy for slowly varying effects; the physical existence of 'Weber waves' cannot be doubted.

Absolute space

The original Weber force law between moving charges, equation (5), is valid only for relative coordinates between two moving charges. When this force law is written in terms of electrostatic and magnetostatic fields, equations (29), (25), and (20), it is valid only in the laboratory frame of reference; as the fields are defined only in terms of the frame of reference of the observing instruments. The Weber theory extended to radiation, as given by equations (41), involves time retardation assuming simply a velocity c between source and observer, in particular, a velocity c relative to the observer. This result (41) does not, therefore, include the effect of absolute space.

The velocity of energy propagation of electromagnetic waves is observed to be fixed relative to absolute space[11–16] and not fixed relative to the moving observer. The result (41) is, thus, only valid for an absolute stationary observer or for v/c negligible.

Absolute space may be introduced by noting that the retarded potentials (41) should be, in fact, defined in terms of the *apparent* retarded effects as observed in the moving laboratory. The velocity c in equations (41) and (42) should, therefore, be interpreted as the phase velocity, $\mathbf{c}'$, observed in the moving laboratory. The wave equations (42) clearly indicate that c must be interpreted as a phase velocity. The phase velocity c' observed in the moving laboratory is, therefore, needed in equations (41) and (42) and not the velocity of energy propagation, $\mathbf{c}^* = \mathbf{c} - \mathbf{v}_o$, where $\mathbf{v}_o$ is the absolute velocity of the observer, or laboratory. As has already been stressed,[17] electromagnetic radiation observed in an absolutely moving system must have *two* velocities and not just *one*, the phase velocity $\mathbf{c}'$ and the velocity of energy propagation $\mathbf{c}^*$. They need not have the same magnitude nor direction.

The original Weber theory is based only on relative coordinates between source and observer; so the effect of absolute space can only be observed when time retardation must be taken into account. To obtain the appropriate mathematical expressions it is sufficient to consider the Voigt–Doppler effect[17,18] for electromagnetic radiation in absolute space. As has been demonstrated[17] the Voigt–Doppler effect explains all of the known observations of the propagation of electromagnetic waves in absolute space, in particular, the observations of Roemer and Bradley,[11] Michelson–Morley,[19] Sagnac,[12] Michelson–Gale,[13] Conklin,[14,15] and Marinov.[16]

For a radiating source moving with the velocity $\mathbf{v}_s$ the propagation constant $\mathbf{k}'$, frequency ω', phase velocity $\mathbf{c}'$, and the velocity of energy propagation $\mathbf{c}^*$ in a system moving with the absolute velocity $\mathbf{v}_o$ along the x axis the Voigt–Doppler effect yields:[17]

$$\begin{aligned}
\mathbf{k}' &= k_s \frac{\gamma_o(c_x - v_o)\mathbf{e}_x + c_y\mathbf{e}_y + c_z\mathbf{e}_z}{c\gamma_s(1 - \mathbf{v}_o\cdot\mathbf{c}/c^2)(1 - \mathbf{v}_s\cdot\mathbf{c}/c^2)} \\
\omega' &= \omega_s\gamma_o(1 - \mathbf{v}_o\cdot\mathbf{c}/c^2)/\gamma_s(1 - \mathbf{v}_s\cdot\mathbf{c}/c^2) \\
\mathbf{c}' &= (c_x - v_o)\mathbf{e}_x + (c_y\mathbf{e}_y + c_z\mathbf{e}_z)/\gamma_o \\
\mathbf{c}^* &= \mathbf{c} - \mathbf{v}_o
\end{aligned} \tag{43}$$

where $\mathbf{e}_x$, $\mathbf{e}_y$, $\mathbf{e}_z$ are unit vectors along cartesian coordinate directions, subscripts s refer to the source and subscripts o refer to the observer, and $\gamma_s = 1/\sqrt{1 - v_s^2/c^2}$ and $\gamma_o = 1/\sqrt{1 - v_o^2/c^2}$. These wave parameters (43) are consistent with equations (41) and (42) where c is replaced by c'.

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